

Supplement to: Spatial patterns of probabilistic temperature change projections from a multivariate Bayesian analysis

R. Furrer

Mathematical and Computer Sciences Department, [Colorado School of Mines](#), Golden, CO 80401, USA

R. Knutti

Climate and Global Dynamics, [National Center for Atmospheric Research](#), Boulder, CO 80305, USA

S. R. Sain

Institute for Mathematics Applied to Geosciences, [National Center for Atmospheric Research](#), Boulder, CO 80305, USA

D. W. Nychka

Institute for Mathematics Applied to Geosciences, [National Center for Atmospheric Research](#), Boulder, CO 80305, USA

G. A. Meehl

Climate and Global Dynamics, [National Center for Atmospheric Research](#), Boulder, CO 80305, USA

We provide a thorough discussion of the statistical method on which our approach is based. The following three sections discuss the model and its parameters, the model assumptions, and, finally, the implementation.

1 Spatial Hierarchical Bayesian Model

In this section we present a spatial hierarchical model proposed by [Furrer et al. \(2006\)](#) to synthesize climate temperature projections based on $N = 21$ high resolution coupled climate models (AOGCMs). The temperature fields from the AOGCMs are stacked into vectors of length n (total number of grid points), and the future temperature climate change is denoted

$$\mathbf{D}_i = \mathbf{Y}_i - \mathbf{X}_i \quad i = 1, \dots, N = 21$$

where \mathbf{X}_i and \mathbf{Y}_i are the temperature fields averaged over the years 1980-2000 and 2080-2100 respectively. The goal is to draw inference about the true but unknown future climate temperature change from the observed fields \mathbf{D}_i . To achieve this, we use a standard hierarchical Bayesian approach. The hierarchical concept decomposes a difficult problem into several layers which are simpler to model and describe. Here we use the typical decomposition into three levels: (1) data, (2) process, and (3) prior.

Data level: The data level models the future temperature change as a spatial process. Consistent with the more traditional decomposition in spatial modeling ([Cressie, 1993](#); [Banerjee et al., 2004](#)) where the mean corresponds to global (first-order) behavior and the error captures local (second-order) behavior, we

assume that the climate change can be expressed as an additive decomposition of a large scale climate signal and small scale signals consisting of model bias and internal model variability. Using mathematical notation we have

$$\mathbf{D}_i = \boldsymbol{\mu}_i + \boldsymbol{\varepsilon}_i \quad \mathbf{D}_i \mid \boldsymbol{\mu}_i, \phi_i \stackrel{\text{iid}}{\sim} \mathcal{N}_n(\boldsymbol{\mu}_i, \phi_i \boldsymbol{\Sigma}) \quad (1)$$

$i = 1, \dots, N$, where \mid means “conditioned on”, \mathcal{N}_n is an n -dimensional normal distribution and where $\stackrel{\text{iid}}{\sim}$ means independently and identically distributed. The spatial correlation matrix $\boldsymbol{\Sigma}$ is specified in advance and $\phi_i > 0$ are scale parameters.

We will assume that the remaining small scale structure $\boldsymbol{\varepsilon}_i$ in equation (1) is an isotropic and stationary processes on the sphere. This means that the small scale structure has constant variance and that for two points (lat_1, lon_1) and (lat_2, lon_2) on the sphere the error covariance between these two points, say $\text{Cov}(\boldsymbol{\varepsilon}_1, \boldsymbol{\varepsilon}_2)$, depends only on the great circle distance θ and not on their latitude and longitude.

To describe the aforementioned covariances, we consider families of isotropic correlation functions on \mathbb{R}^3 restricted to the sphere by replacing the Euclidean distance by $2R \sin(\theta/2)$, where R is the globe’s radius and $\theta \in [0, \pi]$ is the great circle distance. The resulting covariance functions are positive definite on the sphere (see [Yaglom, 1987](#); [Weber and Talkner, 1993](#); [Gaspari and Cohn, 1999](#), and [Gneiting, 1999](#)). Specifically, we use the exponential correlation function

$$c(\theta; \tau) = \eta I_{\{\theta=0\}} + \exp(-2R \sin(\theta/2)/\tau) \quad (2)$$

where $\tau > 0$ is the range parameter dictating the rate at which the correlation decays, I is the indicator

function with $I_{\{\theta=0\}} = 1$ when $\theta = 0$ and 0 otherwise and η accounts for variations that are smaller than the grid size.

Our approach does not include the parameters τ and η in the hierarchical model because of computational complexity. See Section 3 for a discussion on the “empirical Bayes” approach used instead.

Process level: The second hierarchical level models the large scale climate signals $\boldsymbol{\mu}_i$, $i = 1, \dots, N$. We use a dimension reduction technique and assume that the climate signals are a linear combination of basis functions, i.e. $\boldsymbol{\mu}_i = \mathbf{M}\boldsymbol{\theta}_i$, where the given “design” matrix \mathbf{M} contains p basis functions with p much smaller than n , the number of total grid points. The $\boldsymbol{\theta}_i$ are modelled as:

$$\boldsymbol{\theta}_i \mid \boldsymbol{\vartheta}, \psi_i \stackrel{\text{iid}}{\sim} \mathcal{N}_p(\boldsymbol{\vartheta}, \psi_i \mathbf{I}) \quad \psi_i > 0 \quad i = 1, \dots, N$$

where \mathbf{I} is the identity matrix. In other words, $\boldsymbol{\theta}_i$ is distributed around $\boldsymbol{\vartheta}$, the true but unknown parameter. Therefore, the “true” large scale climate change pattern is denoted as $\mathbf{M}\boldsymbol{\vartheta}$. Centering these coefficients vectors about the true climate change coefficients reflects our assumption that the individual climate models do not exhibit systematic large scale errors. However, we do expect departures between each model and the true climate and this variation is captured by the variance term $(\psi_i \mathbf{I})$. In particular the variation of ψ_i across different models reflects different levels of bias and internal variability for a given AOGCM.

The basis functions used to construct the design matrix \mathbf{M} need to be sufficiently flexible to represent the mean structure of the temperature difference fields, and to achieve this goal we use spherical harmonics and indicator functions.

The spherical harmonics are a generalisation of a sine-cosine decomposition of a real valued function to the sphere and provide a complete set of basis functions for the square integrable functions on a sphere. In our approach, we use a truncated set of spherical harmonics consisting of the first 121 functions. The spherical harmonics are continuous functions and it would require many functions to approximate indicator or non-smooth functions. Temperature fields are inherently non-smooth (land, water, sea ice, etc.) and it is more efficient to use a few indicator functions to incorporate this characteristic compared to a large set of spherical harmonics (i.e. smaller parameter space). However, both sets jointly do not form a complete basis. If that would be the case, the small scale process of each model would be identically zero. Incorporating the indicator functions results in the error processes $\boldsymbol{\varepsilon}_i$ exhibiting more stationary and isotropic behavior. The model is very

robust with respect to small changes in the definition of the indicator functions, especially if regions with rather smooth temperature fields are affected. The indicator functions should represent large scale geographical regions and any resemblance with political boundaries are coincidental.

Figure 1 shows four examples of spherical harmonics and Figure 2 shows all of the used indicator functions.

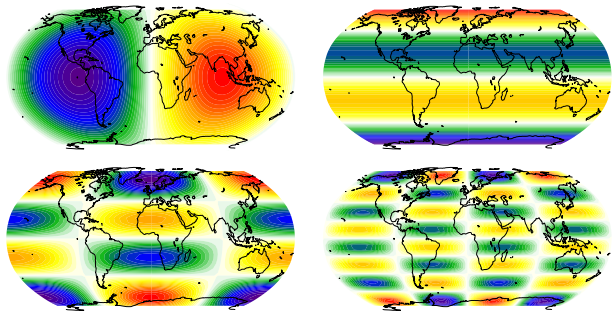


Figure 1: Examples of spherical harmonics. The fields correspond to spherical harmonics with increasing resolution.

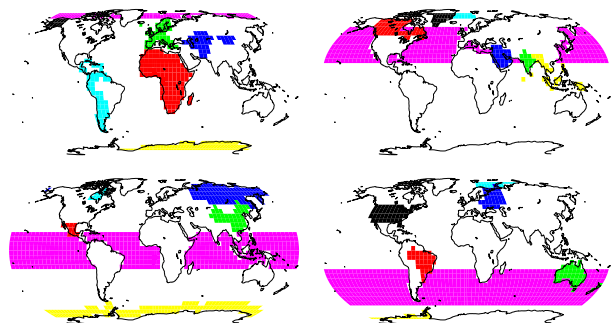


Figure 2: Indicator basis functions. For readability, the basis functions are shown in different panels (each containing 7 different basis function).

Prior level: The last level puts priors on the process parameters.

$$\begin{aligned} \phi_i &\stackrel{\text{iid}}{\sim} \Gamma(\xi_1, \xi_2) & \xi_1, \xi_2 > 0 & \quad i = 1, \dots, N \\ \psi_i &\stackrel{\text{iid}}{\sim} \Gamma(\xi_3, \xi_4) & \xi_3, \xi_4 > 0 & \quad i = 1, \dots, N \\ \boldsymbol{\vartheta} &\sim \mathcal{N}_p(\mathbf{0}, \xi_5 \mathbf{I}) & \xi_5 > 0 & \end{aligned}$$

where Γ denotes the inverse Gamma distribution and where ξ_1, \dots, ξ_5 are hyperparameters. The hyperparameters have to be specified by an expert user and determine the shape of the prior distribution. In general, the posterior distribution is determined by the prior and the likelihood, i.e. more available observations decrease the influence of the prior distribution.

With our model structure, we assume that the design of each AOGCM model is such that it models the truth to the best knowledge of its developers. This implies that the scaling parameters ψ_i should be smaller compared to ϕ_i . The choice of ξ_1, \dots, ξ_4 should therefore enforce this. As it turns out, ξ_1, ξ_2, ξ_3 can be set to any arbitrarily small value (representing no prior information) and ξ_4 should be set between 0.75 and 2. This last parameter determines the weighted global climate change spread. Smaller or larger values result in narrower or wider posterior PDFs with respect to the AOGCMs' spread. We assume that the posterior distribution should have a similar spread as the AOGCM model spread and we set $\xi_4 = 1$ in our setting. Extending the currently used statistical model to account for individual model ensemble members would diminish or almost eliminate the sensitivity of the results on the parameter choice.

Note also that if we include more basis functions in \mathbf{M} then we expect ϕ_i to be smaller. Since we do not have any independent information about the true $\boldsymbol{\theta}$ we choose a large value for ξ_5 , reflecting an uninformative prior and further justifying the identity as its correlation matrix.

2 Model Assumptions

This section lists the most relevant statistical assumptions on which our approach is based and discusses the climatological implications thereof.

The statistical methodology considers the difference fields as unbiased and as independent, which essentially is equivalent to the following two assumptions on the AOGCMs. First, each individual AOGCM is assumed to be unbiased over the entire globe. Second, all AOGCMs are assumed to be independent and are given the same weight. The assumptions are not entirely satisfied in reality. To some degree, all AOGCMs are known to have systematic biases, in particular on small scales. Also, the independence assumption implies that with increasing number of AOGCMs the uncertainty in the climate change estimate decreases (by the central limit theorem). Statistically, it does probably not make sense to think about asymptotics when AOGCM output are the observations. On the other hand, we also believe that the models are in reality not independent and when this is properly taken into account from a statistical point of view, the uncertainty does not vanish with increasing number of AOGCMs (see [Furrer, 2005](#)). At this point it is unclear how to statistically incorporate dependence between AOGCMs into our model. Ideally, our method should also in-

corporate that some models compare more favorably with observations than others. These assumptions do not imply that all the models have the same climate sensitivity, or that they all have the same transient climate response at a particular time and at some scale. We assume that the design of each AOGCM model is such that it models the truth to the best knowledge of its developers. Each AOGCM “approximates” (or models) the truth with different parameterizations, resolved processes etc. (resulting in different climate sensitivity, transient climate response, etc.). The differences are expressed in the individual precisions used for each of the models. Statistically speaking, with one observation (one model run), we cannot determine mean and standard deviation, but only one of the two. We assume a mean and estimate the standard deviation.

Note that the independence assumption along with the absence of observations in the model does not attribute weights to the models and we do not have a bias and a convergence weighting of the models as used by e.g. [Tebaldi et al. \(2005\)](#). Since the fields are assumed to be independent, increasing the number of AOGCMs in the study decreases the uncertainty about the mean climate change. However, many scientists believe that increasing the number of AOGCMs does not increase our knowledge about the future climate change as much of the models are built on the same set of approximations and use fundamentally similar parameterizations for sub-grid processes. However, to date there are no published methodologies on how to account for model dependence in multi-model ensembles. Finally, note that Model (1) together with the independence assumption implies that the density of the entire dataset, conditional on $\boldsymbol{\mu}_i$ and ϕ_i , $i = 1, \dots, N$, is the product of N Gaussian density functions dictated by $\mathcal{N}_n(\boldsymbol{\mu}_i, \phi_i \boldsymbol{\Sigma})$.

Another hypothesis is the assumption concerning the small scale variability and small scale correlation structure. Assuming an stationary and isotropic process implies constant variance and a covariance of a functional form that depends only on the great circle distance. Exploratory graphical analysis (e.g. [Cressie, 1993](#)) shows that with our choice of basis functions the resulting small scale process is close to being isotropic over land or over the ocean only. Only a few models exhibit slightly non-stationary behavior in different regions, like over sea ice or in the intertropical convergence zone (ITCZ). It is true that the hypothesis of a constant error variance is rather strong and implies that we overestimate the variance over the ocean and underestimate it over land. On the other hand, this does not affect the estimation of the climate change signal significantly. It only im-

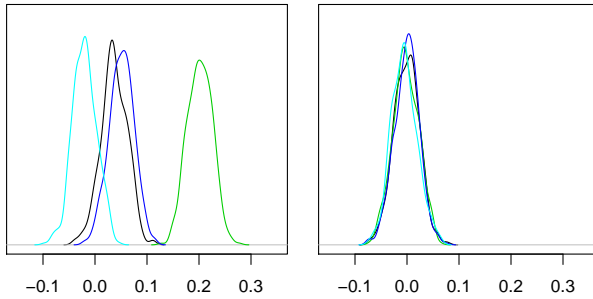


Figure 3: Kernel estimates of the DJF posterior densities for the spherical harmonics given in Figure 1 (left panel) and for spherical harmonics 166 to 169 (right panel). The estimates for the high resolution spherical harmonics are centered around zero (A1B scenario 2080–2100 relative to 1980–2000).

plies that we have a poor description of the bias and internal variability of each of the AOGCMs, which – as mentioned – does not have consequences on the estimation of the large scale signal.

The approach that the true climate signal is represented as a combination of basis functions is not a critical assumption because the chosen basis functions span a large space. There are several ways to test if the basis functions as well as their number are a good choice. First, the basis functions need to be capable of representing the individual difference fields, in other words, the number of basis functions determines the resolution of the large scale structure. A second possibility is post-verification. If too many functions are included, the model may be over-parameterized and the posterior densities of the coefficients of the high resolution spherical harmonics are centered around zero as shown in the right panel of Figure 3. If too few functions are included, the resulting posterior fields may exhibit unrealistic or “blobby” features (see Figure 4). Notice the bulky field when using few functions (top panel). The lower two panels have a similar resolution. However, as we depict only one posterior sample, the fields contain more structure than “averaged” fields shown in Figure 1 of the article. For climate fields with distinct local features (like precipitation fields) more basis functions are required compared to temperature or surface pressure fields. For temperature fields any number between 70 to 200 functions yields virtually identical results with respect to the global or local mean climate changes and 121 basis functions were used in this study.

The authors are working on a much more complicated model setup that includes, among other, several model runs and observational fields. The authors

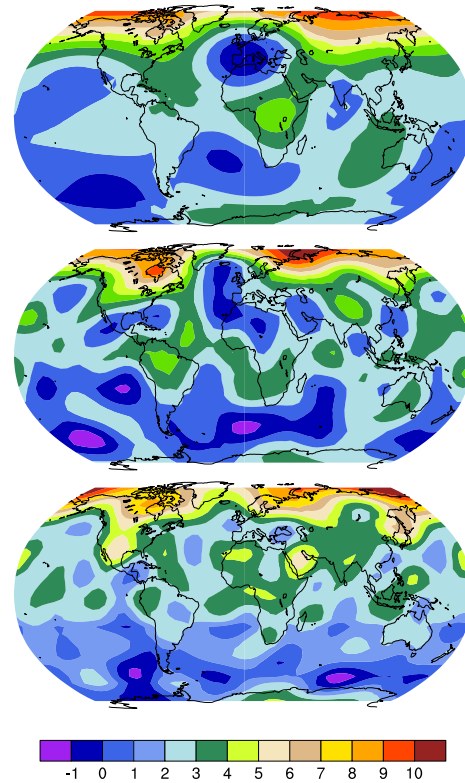


Figure 4: Sample of the DJF posterior mean climate change obtained using 36, 100, and 196 spherical harmonics (top to bottom) in °C by 2080–2100 in the A1B scenario, relative to 1980–2000.

believe that the new statistical model should be able to distinguish and identify model bias and internal variability. Further, observations should guide the choice of the prior parameters, especially of the prior information for the magnitude of ψ_i .

3 Implementation

Our approach does not include the parameters τ and η in the hierarchical model because of computational complexity. The values of the parameters are chosen according to an “empirical Bayes” approach by considering the data model as a linear regression with correlated errors and estimating the error parameters with an iterative procedure.

To begin, the algorithm assumes initial estimates of η , τ and ϕ_i with which a covariance matrix $\phi_i \Sigma$ is constructed, then one estimates the mean structures, i.e. the vector θ_i , via weighted least squares (WLS). Given the estimate $\mathbf{M}\hat{\theta}_i^*$ of the mean $\mathbf{M}\theta_i$, $\mathbf{D}_i - \mathbf{M}\hat{\theta}_i^*$ is used to obtain the estimates $\hat{\eta}$, $\hat{\tau}$ and $\hat{\phi}_i$ of the covariance structure $\phi_i \Sigma^*$ with a method-of-moments approach. Now $\hat{\theta}_i^*$ is updated using WLS

with the covariance matrix $\hat{\phi}_i^* \hat{\Sigma}^*$ and we obtain a second estimate $\hat{\theta}_i^*$. The iteration is carried out until a specified convergence criterion is met. The algorithm generally converges quickly and in most cases, only a few iterations were required.

The fit of the hierarchical model through the Markov Chain Monte Carlo algorithm gives (1) estimates of the true regression coefficients, (2) the uncertainty around them and (3) estimates of the small scale covariance. By recombining the coefficient estimates with the basis functions an estimate of the true climate change field is derived. The uncertainty around this field can be determined, for example, by examining ensembles constructed from draws from the posterior distribution of the coefficient estimates and combining these with the basis functions. Since the model accounts for the spatial correlation of the true climate change through the basis functions, and of the model specific bias and internal variability through the error covariance, the probabilistic projections derived for the entire climate change fields represent the spatially joint probability of climate change for each of the locations. We discuss this now in more detail.

The statistical approach is to obtain the posterior distribution of $\mathbf{M}\boldsymbol{\vartheta}$ given the model observations \mathbf{D}_i , i.e. $\mathbf{M}\boldsymbol{\vartheta} \mid \mathbf{D}_1, \dots, \mathbf{D}_N$. The posterior density can be derived via Bayes' theorem (e.g., [Bernardo and Smith, 1994](#)), synthesized as

$$\begin{aligned} & [\text{process} \mid \text{data, parameters}] \propto \\ & [\text{data} \mid \text{process, parameters}] \cdot \\ & [\text{process} \mid \text{parameters}] \cdot [\text{parameters}] \end{aligned}$$

The densities, denoted by [...], on the right-hand side are given by the three levels of the hierarchical model. The joint posterior on the left-hand side is a complicated distribution from which it is impossible to draw directly. However, the posterior can be sampled using a Markov chain Monte Carlo (MCMC) procedure known as the Gibbs sampler ([Geman and Geman, 1984](#); [Gelfand and Smith, 1990](#)). The essence of the MCMC approach is to simulate complex joint probability distributions by sampling from a Markov chain with a stationary, ergodic distribution that is identical to the posterior distribution (see [Gilks et al., 1996](#); [Robert and Casella, 1999](#)).

Essentially the Gibbs sampler works as follows. For each parameter in the model its distribution conditional on all the other random quantities in the model is identified. Such distributions are called full conditionals because only the parameter of interest is allowed to be random and the entire remaining part of the model is fixed (or conditioned upon). The Monte Carlo algorithm cycles through the parameters by simulating a new value for each parameter

based on the full conditional distribution and the current values of the other parameters. Under weak assumptions the sequence converges to the intended distribution.

Since the hierarchical model is based on multivariate normal and inverse gamma distributions, it is possible to derive the full conditionals in closed form. Starting from the joint density $[\boldsymbol{\vartheta}, \{\boldsymbol{\theta}_i\}, \{\phi_i\}, \{\mathbf{D}_i\} \mid \xi_1, \dots, \xi_5]$, the full conditional distributions are:

$$\begin{aligned} \boldsymbol{\vartheta} \mid \dots &\sim \mathcal{N}_p(\mathbf{A}^{-1}\mathbf{b}, \mathbf{A}^{-1}) \\ \mathbf{A} &= \frac{1}{\xi_5} \mathbf{I} + \sum_{i=1}^N \frac{1}{\psi_i} \mathbf{I} & \mathbf{b} &= \sum_{i=1}^N \frac{1}{\psi_i} \boldsymbol{\theta}_i \\ \boldsymbol{\theta}_i \mid \dots &\sim \mathcal{N}_p(\mathbf{A}^{-1}\mathbf{b}, \mathbf{A}^{-1}) \\ \mathbf{A} &= \frac{1}{\psi_i} \mathbf{I} + \frac{1}{\phi_i} \mathbf{M}^\top \boldsymbol{\Sigma}^{-1} \mathbf{M} \\ \mathbf{b} &= \frac{1}{\psi_i} \boldsymbol{\vartheta} + \frac{1}{\phi_i} \mathbf{M}^\top \boldsymbol{\Sigma}^{-1} \mathbf{D}_i \\ \phi_i \mid \dots &\sim \Pi\left(\xi_1 + \frac{n}{2}, \xi_2 + \frac{1}{2}(\mathbf{D}_i - \mathbf{M}\boldsymbol{\theta}_i)^\top \boldsymbol{\Sigma}^{-1}(\mathbf{D}_i - \mathbf{M}\boldsymbol{\theta}_i)\right) \\ \psi_i \mid \dots &\sim \Pi\left(\xi_3 + \frac{p}{2}, \xi_4 + \frac{1}{2}(\boldsymbol{\theta}_i - \boldsymbol{\vartheta})^\top (\boldsymbol{\theta}_i - \boldsymbol{\vartheta})\right) \end{aligned}$$

with $i = 1, \dots, N$ and where the ‘...’ to the right of the conditioning sign refers to all the random quantities in the model, apart from the parameter to be drawn, and the data. Given the closed form of all the full conditionals, it is straightforward to implement a Gibbs sampler in any numerical software program. We used the freely available computer software R ([Ihaka and Gentleman, 1996](#); [R Development Core Team, 2006](#)). For the numerical experiments presented here, we run the sampler for a total of 20,000 iterations discarding the first half of the simulated values to obtain convergence and saving every 20th draw. Thus, we base our conclusions on a total of 500 values for each parameter, representing a sample from its posterior distribution. On a reasonable desktop PC this task can be performed within a few hours.

The following pseudo algorithm summarizes the Gibbs sampler, as implemented in our study. Table 1 summarizes all the parameters and values used.

```

Program Gibbs sampler
  Load AOGCM data

  Set program parameters
  Construct design matrix and
    initialize auxiliary variables
  Estimate tau and eta
  Construct Sigma and its inverse

  For k from 1 to 20000
    For i from 1 to 21

```

```

    Sample theta_i
Sample vartheta
For i from 1 to 21
    Sample phi_i
For i from 1 to 21
    Sample psi_i

Discard the first 10000 then
    save every 20th sample
End For

Save samples
End Program

```

Table 1: Parameter and hyperparameter specifications for the Gibbs sampler for DJF. For JJA, the same values have been used except for tau $\tau = 0.225$.

Prior parameters:	
$\xi_1, \xi_2, \xi_3, \xi_4, \xi_5$:	0.00, 0.00, 0.00, 1.00, 1.00
Covariance parameters:	
η, τ :	0.00 0.325
Number of basis functions	
spherical harmonics:	121
indicator functions:	28
Gibbs sampler parameters:	
run and discard length:	20,000 10,000
posterior sample length:	500 (every 20th)

References

- Banerjee, S., A. E. Gelfand, and B. P. Carlin (2004), *Hierarchical Modeling and Analysis for Spatial Data*, Chapman & Hall/CRC. 1
- Bernardo, J. M., and A. F. M. Smith (1994), *Bayesian Theory*, John Wiley & Sons Inc., Chichester. 5
- Cressie, N. A. C. (1993), *Statistics for Spatial Data*, John Wiley & Sons Inc., New York, revised reprint, revised reprint of the 1991 edition, A Wiley-Interscience Publication. 1, 3
- Furrer, R. (2005), Covariance estimation under spatial dependence, *J. Multivariate Anal.*, 94(2), 366–381. 3
- Furrer, R., S. R. Sain, D. W. Nychka, and G. A. Meehl (2006), Multivariate bayesian analysis of atmosphere-ocean general circulation models, *Environmental and Ecological Statistics*, in press, also available from <http://www.mines.edu/~rfurrer/research/documents/furrer-et-al.ees.color.pdf>. 1
- Gaspari, G., and S. E. Cohn (1999), Construction of correlation functions in two and three dimensions, *Q. J. R. Meteorol. Soc.*, 125, 723–757. 1
- Gelfand, A., and A. Smith (1990), Sampling based approaches to calculating marginal densities, *J. Amer. Stat. Assoc.*, 85, 398–409. 5
- Geman, S., and D. Geman (1984), Stochastic relaxation, gibbs distributions, and the bayesian restoration of images, *IEEE Trans. Pattern Anal. Mach. Intell.*, 6(6), 721–741. 5
- Gilks, W. R., S. Richardson, and D. J. Spiegelhalter (1996), *Markov Chain Monte Carlo in Practice*, Chapman & Hall. 5
- Gneiting, T. (1999), Correlation functions for atmospheric data analysis, *Q. J. R. Meteorol. Soc.*, 125, 2449–2464. 1
- Ihaka, R., and R. Gentleman (1996), R: A language for data analysis and graphics, *Journal of Computational and Graphical Statistics*, 5(3), 299–314. 5
- R Development Core Team (2006), *R: A language and environment for statistical computing*, R Foundation for Statistical Computing, Vienna, Austria, <http://www.R-project.org>. 5
- Robert, C. P., and G. Casella (1999), *Monte Carlo Statistical Methods*, Springer. 5
- Tebaldi, C., R. L. Smith, D. Nychka, and L. O. Mearns (2005), Quantifying uncertainty in projections of regional climate change: a Bayesian approach to the analysis of multimodel ensembles, *J. Climate*, 18(10), 1524–1540, doi:10.1175/JCLI3363.1. 3
- Weber, R. O., and P. Talkner (1993), Some remarks on spatial correlation function models, *Month. Weath. Rev.*, 121, 2611–2137; Corrigendum: (1999), 127, 576. 1
- Yaglom, A. M. (1987), *Correlation theory of stationary and related random functions. Vol. I*, Springer Series in Statistics, Springer-Verlag, New York. 1

R. Furrer, Mathematical and Computer Sciences Department, **Colorado School of Mines**, Golden, CO 80401, USA. (rfurrer@mines.edu)

R. Knutti, Climate and Global Dynamics, **National Center for Atmospheric Research**, Boulder, CO 80305, USA. (knutti@ucar.edu)

S. R. Sain, Institute for Mathematics Applied to Geosciences, **National Center for Atmospheric Research**, Boulder, CO 80305, USA. (ssain@ucar.edu)

D. W. Nychka, Institute for Mathematics Applied to Geosciences, **National Center for Atmospheric Research**, Boulder, CO 80305, USA. (nychka@ucar.edu)

G. A. Meehl, Climate and Global Dynamics, **National Center for Atmospheric Research**, Boulder, CO 80305, USA. (meehl@ucar.edu)