Separating physics, dynamics and tracer grids in CAM-SE
(Community Atmosphere model - Spectral Elements)

Peter Hjort Lauritzen
National Center for Atmospheric Research
Boulder, Colorado, USA

Collaborators:
M.A. Taylor, P.A. Ullrich,
S. Goldhaber, J. Bacmeister

CMMAP (Center for Multiscale Modeling of Atmospheric Processes) meeting
January 7, 2016
Boulder, Colorado, USA
Getting away from the lat-lon grid ...

CAM=NCAR’s Community Atmosphere Model

- Scalability
- Static mesh-refinement capability
- ...

CAM-FV  (finite volume)
Lin (2004)

CAM-SE  (spectral elements)
Taylor et al., (1997)
Dennis et al., (2012)
CAM-SE: NCAR Community Atmosphere Model with Spectral Elements dynamical core

Continuous Galerkin finite-element method (Taylor et al., 1997) on a cubed-sphere:

- Discretization is mimetic => mass-conservation & total energy conservation
- Conserves axial angular momentum very well (Lauritzen et al., 2014)
- Support static mesh-refinement and retains formal order of accuracy!
- Highly scalable to at least \( O(100K) \) processors (Dennis et al., 2012)
- Competitive “AMIP-climate” (Evans et al., 2012)
- Lower computational throughput for many-tracer applications
- Tracer transport accuracy?
of AAM from the dynamical core (second column) are the same order of magnitude as the physical sources/sinks of AAM (third column).

LAURITZEN ET AL. (2014)

Figure 1. Angular momentum diagnostics for CAM-FV in the Held-Suarez setup (data are from

MPAS results courtesy of Sanghun Park

Lauritzen et al., 2014
CAM-SE: NCAR Community Atmosphere Model with Spectral Elements dynamical core

Continuous Galerkin finite-element method (Taylor et al., 1997) on a cubed-sphere:

- Discretization is mimetic => mass-conservation & total energy conservation
- Conserves axial angular momentum very well (Lauritzen et al., 2014)
- Support static mesh-refinement and retains formal order of accuracy!
- Highly scalable to at least O(100K) processors (Dennis et al., 2012)
- Competitive “AMIP-climate” (Evans et al., 2012)
- Lower computational throughput for many-tracer applications
- Tracer transport accuracy?
Continuous Galerkin finite-element method (Taylor et al., 1997) on a cubed-sphere: Discretization is mimetic, => mass-conservation & total energy conservation. Conserves axial angular momentum very well (Lauritzen et al., 2014). Supports static mesh-refinement and retains formal order of accuracy! Highly scalable to at least O(100K) processors (Dennis et al., 2012). Competitive "AMIP-climate" (Evans et al., 2012). Lower computational throughput for many-tracer applications.

CAM-SE: NCAR Community Atmosphere Model with Spectral Elements dynamical core

Figure 5: Performance of the CESM atmosphere component model on Intrepid (IBM BG/P) when using the CAM-SE, FV or EUL dynamical core, showing the simulated-years-per-day as a function of the number of processing cores. Atmosphere component times taken from a CESM time-slice simulation, coupling the atmosphere (at 0.25° or T341 resolution), the land model (0.25° resolution), and the sea ice and data ocean model (0.1°). The solid black line shows perfect parallel scalability. When using CAM-SE, the CESM achieves near perfect scalability down to one element per processor, running at 12.2 SYPD on 86,400 cores. Figure from Dennis et al. (2012).

We can now perform climate simulations at unprecedented resolutions and we are starting to resolve some meso-scale motion (at which scales the dynamics fundamentally changes character!)

Peter Hjort Lauritzen (NCAR)
Dynamics II
May 30, 2012 8 / 25
CAM-SE: NCAR Community Atmosphere Model with Spectral Elements dynamical core

Continuous Galerkin finite-element method (Taylor et al., 1997) on a cubed-sphere:

- Discretization is mimetic => mass-conservation & total energy conservation
- Conserves axial angular momentum very well (Lauritzen et al., 2014)
- Support static mesh-refinement and retains formal order of accuracy!
- Highly scalable to at least O(100K) processors (Dennis et al., 2012)
- Competitive “AMIP-climate” (Evans et al., 2012)
- Lower computational throughput for many-tracer applications
- Tracer transport accuracy?
• Computational grid: 3 elements, 4 quadrature points in each element (np=4)
• This quadrature will integrate polynomials of degree 3 exactly
• Note: quadrature points are duplicated on element edges
Let the initial condition for GLL point values be a degree 3 polynomial.
• Let the initial condition for GLL point values be a degree 3 polynomial
• The polynomial basis exactly represents initial condition
• Within each element the dynamical core advances one Runga-Kutta step
• Note each element advances the solution in time independently
• Within each element the dynamical core advances one Runge-Kutta step
• Note each element advances the solution in time independently
• Discontinuities may develop at element edges
• Within each element the dynamical core advances one Runga-Kutta step
• Note each element advances the solution in time independently
• Discontinuities may develop at element edges – averaging at element edges
• This process is repeated for every Runge-Kutta stage (currently 5 times per dynamics time-step)

• Physics is “run on GLL grid”
• Physics update: say it perturbs one point value
- Physics update: say it perturbs one point value
- **Polynomial basis changed in element 2**
- **Basis functions only C⁰ at element edges**
Topography smoothing in CAM

30 year AMIP simulations

OMEGA, JJA, model level 16 (approximately 323 hPa)

Notation: $2.5 \text{xdiv} = 2.5^2$ times more divergence damping than vorticity damping
4x, 8x, ..., 32x = smoothing of surface geopotential height

Topography smoothing in CAM

30 year AMIP simulations

Total precipitation rate

How do we (/should we?) couple the dynamical core with sub-grid scale parameterizations (physics)?
Traditionally physics and dynamics grids are collocated

- smoothly varying grid in terms of grid size
- Much higher resolution near poles, however, dynamical core usually has filter in the polar regions to filter out small scales
- Aside: Lat-lon grid is “optimal” for minimizing zonal flow errors! … when grid is no longer zonally aligned errors get rather large …
Traditionally physics and dynamics grids are collocated

If you construct control volumes around the quadrature points so that the area of the control volumes equals the Gaussian quadrature weight (times metric term) then a very anisotropic grid results

Gets “worse” with:

- mesh-refined grids
- increasing polynomial order
Instantaneous Omega near 500 hPa 1/day for a month

- Black line: Large SE volumes
- Blue line: Medium SE volumes
- Red line: Smallest SE volumes
Separate physics-dynamics grids?

Current physics/“coupler” grid

Finite-volume equi-angular gnomonic grid
The assessment includes two dynamical cores that are contrast to CAM towards the model top to describing different orders of inner and outer operators. The operators in the advection scheme (PPM) to avoid the inconsistencies described in Lauritzen (2007) when using cell averages as prognostic variables as in Sherwin 1999, Canuto et al. 2007), where the polynomial order. Rather, the spectral element method uses polynomials to represent the prognostic variables inside each element. The spectral element method is an isentropic version of CAM developed at the Geophysical Fluid Dynamics Laboratory (GFDL) and the NASA Goddard Space Flight Center. The advection scheme is based on the Lin and Rood (1996) method but adapted to non-orthogonal cubed-sphere grids (Putman and Lin 2009). Apart from the vertical coordinate the horizontal momentum equation.

\[
\frac{\partial A}{\partial t} + \nabla \cdot (A \mathbf{v}) = \mathbf{F}_v
\]

\[
\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} = -\nabla p + \nabla \cdot \mathbf{F}_v
\]

\[
\frac{\partial \theta}{\partial t} + \mathbf{v} \cdot \nabla \theta = \nabla \cdot \mathbf{Q}_\theta
\]

\[
\frac{\partial \phi}{\partial t} + \mathbf{v} \cdot \nabla \phi = \nabla \cdot \mathbf{Q}_\phi
\]

Here, \( \mathbf{v} \) is the geostrophic wind, \( p \) is the pressure, \( \mathbf{F}_v \) is the forcing, \( \mathbf{Q}_\theta \) is the heat flux, and \( \mathbf{Q}_\phi \) is the mass flux. The strength of the divergence damping increases making the method elementwise mass-conservative (to element-wise conservative and shape-preserving at the node level). The spectral element method is an isentropic version of CAM (FV), in which the physical variables are computed at GLL nodal values.
The assessment includes two dynamical cores that are contrast to CAM towards the model top to de...
CAM-SE-physgrid configuration

Dynamics: Spectral-element dynamics on Gauss-Lobatto-Legendre (GLL) nodal values

(4x4 GLL point in each element; degree 3 Lagrange polynomials)

Tracer advection: Spectral-element method that is element-wise conservative and shape-preserving at the node level

Physics: Physics columns using cell-averaged state of atmosphere
3.2. Cubed-sphere grid models

The assessment includes two dynamical cores that are contrast to CAM towards the model top to de is the smallest grid cell area in the domain.

An example of a two-dimensional extension volume implementation (i.e., the Lin and Rood, 1996, (c) icosahedral grid based on hexagons and pentagons. The Figure 3: (a) The latitude-longitude grid, (b) the cubed-sphere grid based on an equi-angular central projection and 12 Lauritzen (2007, 2009). Like CAM the polar regions and mid-latitudes. Nevertheless, an model does not apply any digital or FFT properties of the divergence, gradient and curl operators, 

The second cubed-sphere dynamical core is NCAR's (HOMME) (Thomas and Loft 2004, Nair et al. spectral element High-Order Method Modeling Environment (JAMES-D) is a cubed-sphere) is a cubed-sphere applies the same inner and outer operators in terms of the number of cells along a panel side. As an explained in Nair et al. (2005). The resolution is speci
ing different orders of inner and outer operators. The inconsistencies described in Lauritzen (2007) when us-

The second-order divergence damping mechanism and an additional fourth-order divergence damp-

Both a weak second-order divergence damping mech-

The strength of the divergence damping increases making the method elementwise mass-conservative (to non-orthogonal cubed-sphere grids (Putman and Lin 2007, 2009). Apart from the vertical coordinate the DCFV and CAM model design is identical to CAM Rasch 2009). Apart from the vertical coordinate the 

In- 

\[
A = \min \quad \text{FV}
\]

\[
\Delta A = \text{FV}, \quad \text{respectively, where}
\]

\[
CUBED, \quad \text{or}
\]

\[
\Delta t
\]

\[
\text{m}
\]

\[
\text{e}
\]

\[
\text{a}
\]

\[
\text{c}
\]

\[
\text{b}
\]

\[
\text{d}
\]

\[
\text{g}
\]

\[
\text{h}
\]

\[
\text{i}
\]

\[
\text{j}
\]

\[
\text{k}
\]

\[
\text{l}
\]

\[
\text{m}
\]

\[
\text{n}
\]

\[
\text{o}
\]

\[
\text{p}
\]

\[
\text{q}
\]

\[
\text{r}
\]

\[
\text{s}
\]

\[
\text{t}
\]

\[
\text{u}
\]

\[
\text{v}
\]

\[
\text{w}
\]

\[
\text{x}
\]

\[
\text{y}
\]

\[
\text{z}
\]

\[
\text{A}
\]

\[
\text{B}
\]

\[
\text{C}
\]

\[
\text{D}
\]

\[
\text{E}
\]

\[
\text{F}
\]

\[
\text{G}
\]

\[
\text{H}
\]

\[
\text{I}
\]

\[
\text{J}
\]

\[
\text{K}
\]

\[
\text{L}
\]

\[
\text{M}
\]

\[
\text{N}
\]

\[
\text{O}
\]

\[
\text{P}
\]

\[
\text{Q}
\]

\[
\text{R}
\]

\[
\text{S}
\]

\[
\text{T}
\]

\[
\text{U}
\]

\[
\text{V}
\]

\[
\text{W}
\]

\[
\text{X}
\]

\[
\text{Y}
\]

\[
\text{Z}
\]

\[
\text{\Delta}
\]

\[
\text{\n}
\]

\[
\text{\times}
\]

\[
\text{\n}
\]

\[
\text{\n}
\]

\[
\text{\n}
\]

\[
\text{\n}
\]

\[
\text{\n}
\]

\[
\text{\n}
\]

\[
\text{\n}
\]

\[
\text{\n}
\]

\[
\text{\n}
\]

\[
\text{\n}
\]

\[
\text{\n}
\]

\[
\text{\n}
\]

\[
\text{\n}
\]

\[
\text{\n}
\]

\[
\text{\n}
\]

\[
\text{\n}
\]

\[
\text{\n}
\]

\[
\text{\n}
\]

\[
\text{\n}
\]

\[
\text{\n}
\]

\[
\text{\n}
\]

\[
\text{\n}
\]

\[
\text{\n}
\]

\[
\text{\n}
\]

\[
\text{\n}
\]

\[
\text{\n}
\]

\[
\text{\n}
\]

\[
\text{\n}
\]

\[
\text{\n}
\]

\[
\text{\n}
\]

\[
\text{\n}
\]

\[
\text{\n}
\]

\[
\text{\n}
\]

\[
\text{\n}
\]

\[
\text{\n}
\]

\[
\text{\n}
\]

\[
\text{\n}
\]

\[
\text{\n}
\]

\[
\text{\n}
\]

\[
\text{\n}
\]

\[
\text{\n}
\]

\[
\text{\n}
\]

\[
\text{\n}
\]

\[
\text{\n}
\]

\[
\text{\n}
\]

\[
\text{\n}
\]

\[
\text{\n}
\]

\[
\text{\n}
\]

\[
\text{\n}
\]

\[
\text{\n}
\]

\[
\text{\n}
\]

\[
\text{\n}
\]

\[
\text{\n}
\]

\[
\text{\n}
\]

\[
\text{\n}
\]

\[
\text{\n}
\]

\[
\text{\n}
\]

\[
\text{\n}
\]

\[
\text{\n}
\]

\[
\text{\n}
\]

\[
\text{\n}
\]

\[
\text{\n}
\]

\[
\text{\n}
\]

\[
\text{\n}
\]
CAM-SE-physgrid configuration

Should we run physics and dynamics on the same resolution grids? Coarser? Finer?

Physics: Coarser, same or finer resolution cell-average grid
The assessment includes two dynamical cores that are contrast to CAM is the smallest grid cell area in the domain. In, e.g., Ullrich et al. (2009).

The standard Space Flight Center. The advection scheme is based on the Lin and Rood (1996) method but adapted to non-orthogonal cubed-sphere grids (Putman and Lin 2007, 2009). Like CAM Rasch 2009). Apart from the vertical coordinate the model does not apply any digital or FFT filtering in.

The strength of the divergence damping increases in terms of the number of cells along a panel side. As an example, 90 cells along each side of a cubed-sphere face explain different orders of inner and outer operators. The operators in the advection scheme (PPM) to avoid the external-mode damping coefficients by subtracting the external-mode damping coefficients from the various operators in the vector momentum equation.

We need to transfer data to and from dynamics-physics grids!!!

Tracer advection: Spectral-element method

Physics: Coarser, same or finer resolution cell-average grid

Dynamics: Spectral-element dynamics on Gauss-Lobatto-Legendre (GLL) nodal values

(4x4 GLL point in each element; degree 3 Lagrange polynomials)
The assessment includes two dynamical cores that are contrast to CAM towards the model top to de is the smallest grid cell area in the domain.

Figure 3: (a) The latitude-longitude grid, (b) the cubed-sphere grid based on an equi-angular central projection and

The strength of the divergence damping increases making the method elementwise mass-conservative (to

ISEN, the cubed-sphere FV . In-

\[ \Delta FV \]

by subtracting the external-mode damping coef-

\[ \text{min} \]

\[ \text{respective, where} \]

\[ \text{CUBED} \text{dy-} \]

\[ \text{∆} \text{FV} \text{, the GEOS}\]

\[ 1 \times \text{CUBED} \text{physgrid} \text{∆} \]

\[ 0 \times 1 \text{layer sponge}. \]

\[ 0.05 \times 1 \text{layer sponge}. \]

\[ 0.02 \times 1 \text{layer sponge}. \]

\[ FV \text{, the GEOS}\]

\[ \text{∆} \text{FV} \text{, r e s p e c t i v e l y , w h e n e r e s e s} \]

\[ \text{fi} \text{cients} \text{a 3-layer sponge}. \]

\[ \text{CUBED) is a cubed-sphere}\]

\[ \text{∆} \text{min} \]

\[ 2 / \text{∆} \text{FV} \text{, the GEOS}\]

\[ \text{∆} \text{FV} \text{, t h e GEOS}\]

\[ \text{∆} \text{min} \]

\[ 0 \text{, respective, where} \]

\[ 0 \text{, respective, where} \]

\[ \text{CUBED) applies the same inner and outer}\]

\[ \text{f inite element method uses}\]

\[ 4 \times 4 \text{GLL point in each element;}\]

\[ \text{degree 3 Lagrange polynomials)}\]

\[ 4 \times 4 \text{GLL point in each element;}\]

\[ \text{tr triangular} \text{nodal values}\]

\[ \text{Lobafo}\]

\[ \text{degree 3 Lagrange polynomials)}\]

\[ 4 \times 4 \text{GLL point in each element;}\]

\[ \text{Legendre (GLL) nodal values}\]

\[ \text{Dynamics on Gauss-}\]

\[ \text{Nota}^{\text{on}}:\ \text{Coarser, same or finer}\]

\[ \text{resolution cell-average grid}\]

\[ \text{Tracer advection}: \text{Spectral-element method}\]

\[ \text{Physics}: \text{Coarser, same or finer}\]

\[ \text{resolution cell-average grid}\]
Separating physics and dynamics grids was a major software engineering task in CAM – affected many parts of the code:

- history (output)
- initialization/restart
- Some parameterizations assumed grids were collocated
Interpolator properties: passing state to physics and returning tendencies to dynamics

- Conservation (coupled climate modeling)
- Shape-preservation (in particular, no negatives)
- Preserve tracer correlations (important for coupling with chemistry)
- Consistent (preserves a constant)
- Other? Total energy?

Implementation constraints/limitations (not “physical” limitations):

- Physics-grid must be a sub-grid of the element
  With some extra software engineering we can relax this constraint!
  (example application: mesh-refinement)

- To reduce MPI communication no halo exchange for physics-dynamics coupling except for boundary exchange at end of interpolation
  (could also be relaxed at the expense of computational cost)
Passing state \((v,T,q,...)\) to physics:

For conservation we interpolate \(dp^*u, dp^*T, dp^*q\)
Passing state \((v,T,q,...)\) to physics:

For conservation we interpolate \(dp^*u, dp^*T, dp^*q\)

Integrate continuous basis functions in each control volume. Conservation and consistency are enforced via a least squares projection onto the space of conservative and consistent maps

!!! this approach is high-order!!!

Ulrich and Taylor (2015)
**Passing state (v,T,q,...) to physics:** For conservation we interpolate \( dp^*u, dp^*T, dp^*q \)

- Interpolation matrix can be pre-computed (it is a linear map)!!!
- After application of interpolation matrix there is a boundary exchange that averages point values on the element boundaries!

Ullrich and Taylor (2015)
Passing state \((v,T,q,...)\) to physics: basis functions oscillatory!

Given GLL point values, \(U_{j,k}(t) = \{0,0,1,0\}\) for \(k=0,...,3\), the Lagrange “reconstruction” is shown on the Figure below:
Monotonicity is enforced via a two-step procedure.

- instead of the regular FEM basis functions we use a set of monotone basis functions (ones whose range is \([0,1]\)).
- This would be sufficient except for the fact that the least squares projection onto conservative/consistent maps could produce some (small) negative values in the mapping coefficients. To fix that problem we then “linearly interpolate” between the conservative/consistent map and the simplest first-order conservative/consistent/monotone map. This has roughly the effect of “borrowing mass” from other GLL nodes within the element.

Ullrich and Taylor (2015)
Monotone linear map

Potential problem: a monotone linear map that does not have any knowledge of the GLL values (i.e. not flow dependent) can at most be 1\textsuperscript{st} order!

Modification to Ullrich-Taylor algorithm:

Since any linear combination of linear maps is conservative and consistent one may “optimally” blend the maps for shape-preservation (“FCT-like method”)
The "FCT" version of Ullrich-Taylor algorithm involves the following steps:

- np*nc columns
- np*nc GLL point values
- nc*nc physics grid values

Mathematically, this can be expressed as:

\[ A_{\text{non-mono}} \times \text{GLL} = \text{PHYS}_{\text{non-mono}} \]
\[ A_{\text{mono}} \times \text{GLL} = \text{PHYS}_{\text{mono}} \]

And for the combined physics, the equation is:

\[ [\alpha A_{\text{mono}} + (1-\alpha) A_{\text{non-mono}} \times \text{GLL}] = \text{PHYS}_{\text{mono}} \]

Where \( \alpha \) is calculated as:

\( \alpha = \frac{\max(\text{GLL}) - \text{PHYS}_{\text{non-mono}}}{\text{PHYS}_{\text{mono}} - \text{PHYS}_{\text{non-mono}}} \) or

\( \alpha = \frac{\min(\text{GLL}) - \text{PHYS}_{\text{non-mono}}}{\text{PHYS}_{\text{mono}} - \text{PHYS}_{\text{non-mono}}} \)
Dynamics to physics grid mapping

Properties we are looking for: Preserve smooth fields and at the same time not generate new extrema for rough distributions (and be mass-conservative and consistent)
Smooth field ("spherical harmonic")

1st order monotone map (not flow dependent): see grid
Smooth field ("spherical harmonic")

NE5NP4 to NC3 (6 degrees global resolution)

Optimally blend conservative and monotone map
Rough field ("slotted cylinder")

Non-monotone conservative

NE5NP4 to NC3 (6 degrees global resolution)
Rough field ("slotted cylinder")

Optimally blend conservative and monotone map
Passing tendencies ($f_v, f_T, f_q, ...$) to dynamics: Use a 1st-order, shape-preserving, conservative linear map
CAM4 forcing: Aqua-planet

Atmospheric model with complete parameterization suite
Idealized surface: no land (or mountains), no sea ice
specified global sea surface temperatures everywhere

=> Free motions, no forced component

Why CAM4? More resolution sensitivity than CAM5 (and it is cheaper!)
Configurations

Data mapped to 3° lat-lon grid for analysis

Length of simulations: 30 months
Min/max moisture forcing

![Graph showing min/max moisture forcing for different models.](image)
Time averaged PS

NE30NP4_APE

Surface pressure

Pa

NE30NP4NC2_APE

Surface pressure

Pa

Difference

NE30NP4NC3_APE

Surface pressure

Pa

NE30NP4NC4_APE

Surface pressure

Pa

difference

996 1000 1004 1008 1012 1016 1020

-4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10
Zonal-time averaged PS

Courtesy of M. Taylor
$Q = \text{Specific humidity}$
RELHUM = Relative humidity
Zonal-time averaged total precipitation rate

Williamson (2004)
PDF
PRECT (30 month simulation - 6h data)

Data mapped to 3° lat-lon grid

Precipitation (mm/day)

Fraction

- NE30NP4NC4_APE
- NE30NP4NC3_APE
- NE30NP4NC2_APE
- NE30NP4_APE
Stationary grid scale forcing
Note that physics grid averages/moves fields away from boundary of element where the solution is least smooth (in element interior the polynomials are $C^\infty$).
Held-Suarez with topography

Vertical velocity at 500 mbar pressure surface

NE30NP4NC2
Held-Suarez with topography

Vertical velocity at 500 mbar pressure surface

Pa/s
CAM-SE: NCAR Community Atmosphere Model with Spectral Elements dynamical core

Continuous Galerkin finite-element method (Taylor et al., 1997) on a cubed-sphere:

- Discretization is mimetic => mass-conservation & total energy conservation
- Conserves axial angular momentum very well (Lauritzen et al., 2014)
- Support static mesh-refinement and retains formal order of accuracy!
- Highly scalable to at least O(100K) processors (Dennis et al., 2012)
- Competitive “AMIP-climate” (Evans et al., 2012)
- Lower computational throughput for many-tracer applications
- Tracer transport accuracy?
The terminator ‘toy’-chemistry test: A simple tool to assess errors in transport schemes
(Lauritzen et al., 2015, GMD)

See: http://www.cgd.ucar.edu/cms/pel/terminator.html

Non-linear
Terminator ‘toy’ chemistry:

$$Cl_2 \rightarrow Cl + Cl : k_1$$
$$Cl + Cl \rightarrow Cl_2 : k_2$$

Exact solution: $Cl + 2*Cl_2 = \text{constant}$

Errors are due to non-conservation of linear correlations usually caused by the limiter/filter and/or physics-dynamics coupling!
A way to accelerate tracer transport:

Basic formulation: Lauritzen et al. (2010), Erath et al. (2013), Erath et al. (2012)

Conservative Semi-Lagrangian Multi-tracer (CSLAM)

Finite-volume Lagrangian form of continuity equation for air (pressure level thickness, $\Delta p$), and tracer (mixing ratio, $q$):

$$\int_{A_k} \psi_{k}^{n+1} dA = \int_{a_k} \psi_{k}^{n} dA = \sum_{\ell=1}^{L_k} \left[ \sum_{i+j \leq 2} c_{i,j}^{(i,j)} w_{k,\ell}^{(i,j)} \right], \quad \psi = \Delta p, \Delta p q,$$

where $n$ time-level, $a_{k,\ell}$ overlap areas, $L_k$ overlap areas, $c^{(i,j)}$ reconstruction coefficients for $\psi_{k}^{n}$, and $w_{k,\ell}^{(i,j)}$ weights.
A way to accelerate tracer transport:

Conservative Semi-Lagrangian Multi-tracer (CSLAM)

**Basic formulation**

Lauritzen et al. (2010), Erath et al. (2013), Erath et al. (2012)

**CSLAM**

- 5-step restriction: flow deformation (upstream area must be simply connected)

**Current implementation in CAM-SE:** CN < 1, where Courant number.

**Spectral-element advection:** RK2 with CN<0.3

=> 3 times longer time-step with CSLAM compared to SE advection scheme
A way to accelerate tracer transport:

Conservative Semi-LAgrangian Multi-tracer (CSLAM)

**Basic formulation**
Lauritzen et al. (2010), Erath et al. (2013), Erath et al. (2012)

**MPI communication**
For every 30 minute physics time-step:

- SE performs 6 tracer time-steps with 2 Runga-Kutta stages => 12 MPI calls
- CSLAM performs 2 tracer time-steps (CN<1) => 2 MPI calls

That said, CSLAM needs a much larger halo than SE.
A way to accelerate tracer transport:

**Basic formulation** Harris et al. (2010)

 Flux-form CSLAM ≡ Lagrangian CSLAM

\[
\int_{A_k} \psi_{k}^{n+1} dA = \int_{A_k} \psi_{k}^{n} dA - \sum_{\epsilon=1}^{4} s_{k\ell}^{e} \int_{a_k^{\epsilon}} \psi dA, \quad \psi = \Delta p, \Delta p q.
\]

where

- \(a_k^{e}\) = ‘flux-area’ (yellow area) = area swept through face \(\epsilon\)
- \(s_{k\ell}^{e}\) = 1 for outflow and -1 for inflow.

Flux-form and Lagrangian forms of CSLAM are equivalent (Lauritzen et al., 2011).
4. Consistency

The continuity equations for air and tracers are coupled:

\[ \int_{A_k} \Delta p_k^{n+1} dA = \int_{a_k} \delta p_k^n dA, \]  \hspace{1cm} (2)

\[ \int_{A_k} (\Delta p q)_k^{n+1} dA = \int_{a_k} (\delta p_k q)^n dA. \]  \hspace{1cm} (3)

If \( q = 1 \) then (3) should reduce to (2).
The continuity equations for air and tracers are coupled:

\[
\int_{A_k} \Delta p_k \, dA = \int_{a_k} \delta p^n_k \, dA, \quad (2)
\]

\[
\int_{A_k} (\Delta p q)_{n+1} \, dA = \int_{a_k} (\delta p_k q)^n \, dA. \quad (3)
\]

If \( q = 1 \) then (3) should reduce to (2).

We need to couple without violating mass-conservation, shape-preservation, and consistency.
4. Consistency

Find upstream area, $a_k$, so that CSLAM predicted mass field is equal to CAM-SE predicted mass field:

$$\Delta p_k^{n+1} (\text{CAM-SE}) = \frac{1}{\Delta A} \int_{a_k} \delta p_k^n \, dA \ (\text{CSLAM}),$$

(4)

If we choose to move departure points around so that (4) is fulfilled a global iteration problem results!

(and I am not sure it is well-posed!)
Basic formulation

Coupling problem formulation

We need to find a departure grid so that

\[ p_{\text{CSLAM}}(x) = p_{\text{SE}}(x) \]  

(3)

⇒ requirements 1-3 are fulfilled with existing CSLAM technology.

Solution: Cast problem in flux-form

Spectral-element method does not operate with fluxes: Taylor et al. have derived a method to compute fluxes, \( \mathcal{F}^{(SE)} \), through the CSLAM control volume faces! presented at ICMS conference in March, 2015.
Basic formulation

Coupling problem formulation

We need to find a departure grid so that

\[ p_{\text{CSLAM}} = p_{\text{SE}} \] (3)

⇒ requirements 1-3 are fulfilled with existing CSLAM technology.

(a) (b)

Figure: Global iteration problem / and it is ill-conditioned since any non-divergent perturbation of points yields the same solution

Peter Hjort Lauritzen (NCAR)
CAM-SE-CSLAM
June 17, 2015 11 / 20

Requirements (desirable properties) for transport schemes

Global climate & climate chemistry modeling

4. Consistency

Find upstream area, \( a_k \), so that CAM-SE predicted mass field:

\[ p_{n+1}^k(\text{CAM-SE}) = \int_a^k p_{n}^k dA(\text{CSLAM}), \] ...

Many details of algorithm (well-posedness, ...) are left out here ...

Peter Hjort Lauritzen (NCAR)
CAM-SE-CSLAM
September 17, 2015 4 / 8

If we choose to move departure points around so that (4) is fulfilled a global iteration problem results! (and I am not sure it is well-posed!)

Solution: Cast problem in flux-form

Given \( \mathcal{F}^{(SE)} \) find swept areas, \( \delta \Omega \), so that:

1. \[ \mathcal{F}^{(CSLAM)} = \int_{\delta \Omega} \Delta p(x, y) dA = \mathcal{F}^{(SE)} \quad \forall \delta \Omega. \]

2. The sum of all the swept areas, \( \delta \Omega \), span the domain without cracks or overlaps
Solution: Cast problem in flux-form

Given $F^{(SE)}$ find swept areas, $\delta \Omega$, so that:

1. $F^{(CSLAM)} = \int_{\delta \Omega} \Delta p(x, y) \, dA = F^{(SE)} \quad \forall \, \delta \Omega.$

2. The sum of all the swept areas, $\delta \Omega$, span the domain without cracks or overlaps
Solution: Cast problem in flux-form

Consistent SE-CSLAM algorithm: step-by-step example

Well-posed? As long as flow deformation \(|\frac{\partial u}{\partial x}| \Delta t \lesssim 1\) (Lipschitz criterion)
Basic formulation

Coupling problem formulation

We need to find a departure grid so that

$$p(CSLAM) = p(SE)$$

⇒ requirements 1-3 are fulfilled with existing CSLAM technology.

Figure: Global iteration problem and it is ill-conditioned since any non-divergent perturbation of points yields the same solution

Requirements (desirable properties) for transport schemes

Global climate & climate chemistry modeling

4. Consistency

Find upstream area, $$a_k$$, so that CSLAM predicts mass field:

$$p_n +_1 k (CAM-SE) = A a_k p_n k dA (CSLAM)$$

Many details of algorithm (well-posedness, ...) are left out here...

If we choose to move departure points around so that (4) is fulfilled a global iteration problem results!

(Solution: Cast problem in flux-form)

Consistent SE-CSLAM algorithm: flow cases
4. Consistency

Find upstream area, $a_k$, so that CSLAM predicted mass field is equal to CAM-SE predicted mass field:

$$\Delta p_k^{n+1}(\text{CAM-SE}) = \frac{1}{\Delta A} \int_{a_k} \delta p_k^n dA \ (\text{CSLAM}),$$

(4)

Local iteration problem to find equivalent upstream areas:
CAM-SE-CSLAM

A new model configuration based on CAM-SE:

- **SE**: Spectral-element dynamical core solving for $\vec{v}$, $T$, $p_s$
  (Dennis et al., 2012; Evans et al., 2012; Taylor and Fournier, 2010; Taylor et al., 1997)

- **CSLAM**: Semi-Lagrangian finite-volume transport scheme for tracers
  (Lauritzen et al., 2010; Erath et al., 2013, 2012; Harris et al., 2010)

- **Phys-grid**: Separating physics and dynamics grids, i.e. ability to compute physics tendencies based on cell-averaged values within each element instead of quadrature points

Lauritzen, Taylor, Overfelt, Ullrich and Goldhaber (2016, IN PREP)
3 tracers: initial conditions

Gaussian “ball”

Zonally symmetric (smooth)

Slotted cylinder
Predictability limit for flow is approximately 12 days (Jablonowski and Williamson, 2006)
CAM-SE-CSLAM

day 9

Average column integrated Cl

4.0E-6 kg/kg

Average column integrated Cl2

4.0E-6 kg/kg

Surface pressure implied by CSLAM

hPa

CONSTANT FIELD - VALUE IS 1

NCAR
ntask 256, 1 degree (NE30NP4NC3), Yellowstone computer

- SE: Total tracers
- CSLAM: Total tracers
- CSLAM: fill halo
- CSLAM: reconstruction
- CSLAM: remap
- CSLAM: high-order weights
- CSLAM: iterate
Performance

1 degree configuration (NE30NP4NC3), 40 tracers

Total tracers time in seconds vs. number of processors.

- CSLAM
- SE

One element per processor.
References


