Introduction

For some time the atmosphere component of the CESM (Community Earth System Model), called CAM (Community Atmospheric Model), has supported a Spectral-Element (SE) dynamical core option; the SE-dynamical core is based on a continuous Galerkin finite-element method discretized on the geostrophic cubed-sphere and it supports static mesh-refinement (see Figure).

Figure 1. (a) Pressure-restore against critical dry-rotor speed that define the elements in CAM-SE. The red dot (top) is the critical flat-rotor speed, and the blue dot (bottom) is the critical air speed for the minimum dry-rotor speed. (b) Energetic mesh refinement in CAM-SE.

Dry-mass vertical coordinate

Consider a general terrain following vertical coordinate 

\[ h(x,y,z) = h(M_{\alpha}) \]

where \( M_{\alpha} \) is dry air mass in a column (per unit area) and

\[ h(M_{\alpha}(\theta)) = 1 \quad \text{and} \quad h(M_{\alpha}(\theta^*)) = 0 \]

where \( M_{\alpha} \) is dry air mass per (unit area) above model top.

Model interface levels (index k=1/2) are defined as

\[ M_{\alpha}(\theta_{k}) = A_{k}^{\alpha} M_{\alpha}(\theta_{k-1}) + B_{k}^{\alpha} M_{\alpha}(\theta_{k}) \]

where \( A_{k}^{\alpha} \) and \( B_{k}^{\alpha} \) are the usual hybrid coefficients.

Note that by removing superscript (d) from the equations above (so that the dry variables represent moist variables), then the vertical coordinate is the usual hybrid-pressure coordinate widely used in hydrostatic global modeling.

Condensate loading

Define set of components of moist air

\[ L_{\alpha}(\theta) = \{v, w, v, e, w, v, w, w, w, w, w\} \]

referring to dry air, water vapor, cloud liquid, cloud ice, rain, and snow. Including condensates in the equations of motion has the following consequences:

- **Hydrometeor equation** includes condensates

\[ p(M) = \rho p(\theta) \int_{0}^{\infty} \rho d\theta \]

where \( \rho \) is gravitational acceleration, \( p(\theta) \) is the density of dry air and total density is

\[ \rho = \rho(\theta) + \rho_\text{vap}(\theta) \]

where \( \rho_v \) is (dry) mixing ratio for component of moist air.

- **Energy conversion term in the thermodynamic equation**

\[ \frac{\partial T}{\partial t} = \nabla \cdot (\mathbf{v} T) - \nabla \cdot (\mathbf{v} T) + \nabla \cdot (\mathbf{v} T) = 0 \]

where \( \mathbf{v} \) is vertical pressure velocity, \( T \) temperature and

\[ c_p = \sum_{\alpha} c_{p,\alpha} \frac{\rho_{\alpha}(\theta)}{\rho(\theta)} \]

where \( c_{p,\alpha} \) is the heat capacity of component \( \alpha \) at constant \( p \), includes condensates. Same for pressure-gradient force.

- **Total energy equation includes condensates**

Integrate adiabatic and frictionless equations of motion over the entire domain

\[ \frac{\partial}{\partial t} \int_{\eta}^{\theta} \rho h d\eta = \int_{\eta}^{\theta} \mathbf{v}_\alpha \cdot \nabla \eta \cdot \mathbf{x} + \nabla \cdot \eta \cdot \mathbf{v} \cdot \Phi \cdot \mathbf{x} \cdot d\eta d\mathbf{x} = 0 \]

where \( \eta = 0.3 L^2 \) is kinetic energy (per unit mass), and \( \Phi \) is surface geopotential.

- **Modified hyperviscosity**

Hyperviscosity applied on approximate pressure surfaces

\[ \nu_{\text{hyp}} = \nabla \cdot (\nu_{\text{hyp}} \nabla \mathbf{v}) - \nabla \cdot (\nu_{\text{hyp}} \nabla \mathbf{v}) \]

and for \( \Delta h^2 \) (dry layer mass) only apply hyperviscosity to difference between \( M_{\alpha}^0 \) and reference dry layer mass.

Reduced damping

Viscosity coefficients have been reduced significantly:

- CAM-SE
- CAM-HOMME
- CAM-SE

**Improved performance**

Throughput in terms of simulated years per day for CAM6 Aqua-planet including standard I/O as a function of number of nodes on NCAR’s Cheyenne supercomputer. Note that for the right-most data-points there is only 9 physics columns per processor.

CAM6 Aqua-planet simulation

Figure 4. 5-year average zonally averaged total precipitation rate as a function of latitude for CAMs Aqua-planet simulations:

- CAM-SE
- CAM-HOMME
- CAM-SE

Figure 5. Total kinetic energy spectrum of the horizontal winds at the 200 hPa level in CAM-HOMME and CAM-SE at 1 horizontal resolution, computed as the mean spectra from 30 days of 6-hourly instantaneous spectra. Black line is the \( k^* \) reference scaling, where \( k \) is spherical wave-number.

**Optional finite-volume tracer transport and finite-volume physics grid**

Tracer transport with CLAM (Conservative Semi-Lagrangian Multi-tracer) scheme consistently coupled to SE (Mon. Wea. Rev. paper: DOI: 10.1175/MWR-D-14-0264.1).

State passed to physics is integrated over (CLAM) finite-volumes (manuscript in preparation).

Reference