NCAR Global Model Topography Generation Software for Unstructured Grids

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Abstract. It is the purpose of this paper to document the NCAR global model topography generation software for unstructured grids. Given a model grid, the software computes the fraction of the grid box covered by land, the gridbox mean elevation, and associated sub-grid scale variances commonly used for gravity wave and turbulent mountain stress parameterizations. The software supports regular latitude-longitude grids as well as unstructured grids; e.g. icosahedral, Voronoi, cubed-sphere and variable resolution grids.

As an example application and in the spirit of documenting model development, exploratory simulations illustrating the impacts of topographic smoothing with the NCAR-DOE CESM (Community Earth System Model) CAM5.2-SE (Community Atmosphere Model version 5.2 - Spectral Elements dynamical core) are shown.

1 Introduction

Accurate representation of the impact of topography on atmospheric flow is crucial for Earth system modeling. For example, the hydrological cycle is closely linked to topography and, on the planetary scale, waves associated with the mid-latitude jets are very susceptible to the effective drag caused by mountains (e.g., Lott and Miller [1997]). Despite the fact that surface elevation is known globally with a high level of precision, the representation of its impact on atmospheric flow in numerical models remains a challenge.

When performing a spectral analysis of high resolution elevation data (e.g., black line in Figure 1), it is clear that Earth’s topography decreases quite slowly with increasing wave number (see also Balmino 1993, Uhrner 2001, Gagnon et al. 2006). Consequently, at any practical model resolution there will always be a non-negligible spectral component of topography present near the grid scale and there will always be a non-negligible spectral component of topography below the grid-scale (sub-grid-scale component). The resolved scale component is the mean elevation in each grid box, \( h \), or, equivalently, the surface geopotential \( \Phi_s \). It is common practice not to force the highest wave numbers directly in the model to alleviate obvious spurious noise (e.g., Navarra et al. 1994, Lander and Hoskins 1997). Hence \( \Phi_s \) is usually smoothed. Figure 1 shows the power spectrum for surface elevation for different levels of smoothing of topography in the NCAR-DOE CESM (Community Earth System Model) CAM (Community Atmosphere Model; Neale et al. 2010), SE (Spectral Element; Thomas and Loft 2005, Dennis et al. 2005) and CAM-FV (Finite-Volume; Lin 2004). Figure 2 shows the associated elevations for a cross section through the Andes mountains. The amount of smoothing necessary is intrinsically linked to the numerical methods and discretization choices in the dynamical core. Further discussion on \( \Phi_s \) smoothing is given in Section 3.2.

The component of topography that cannot be represented by \( \Phi_s \) is referred to as sub-grid-scale topography. Sub-grid-scale processes may be sub-grid flow blocking, flow splitting, sheltering effects, generation of turbulence by roughness and gravity wave breaking (e.g., Bougeault et al. 1990). These processes can have an important impact on the resolved scale flow. Global models usually have a parameterization for gravity wave drag (GWD) (e.g., Eckermann and Chun 2003) and turbulent effects referred to as turbulent mountain stress (TMS). An increasing number of models also have a low-level blocking parameterization (e.g., Lott and Miller [1997], Webster et al. 2003, Zadra et al. 2003, Kim and Doyle 2005, Scinocca and McFarlane 2006) and incorporate the effects of sub-grid-scale topographic anisotropy (i.e. the existence of...
Figure 1. Log/log plot of spectral energy versus wave number $K$ for the ‘raw’ 1km USGS data (GTOPO30), different levels of smoothing for 100km CAM-SE topography, and CAM-FV. Labels ‘04x’, ‘08x’, ‘16x’ and ‘32x’ CAM-SE, refer to different levels of smoothing, more precisely, four, eight, sixteen and thirty two applications of a ‘Laplacian’ smoothing operator in CAM-SE, respectively. Label ‘CAM-FV’ refers to the topography used in CAM-FV at $0.9^\circ \times 1.25^\circ$ resolution. ‘0x’ CAM-SE is the unsmoothed topography on an approximately $1^\circ$ grid CAM-SE grid. Note that the blue (4x, CAM-SE) and brown (CAM-FV) lines are overlaying. Solid straight line shows the $K^{-2}$ slope. The associated surface elevations are shown on Figure 2.

Figure 2. Surface elevation in kilometers for a cross section along latitude $30^\circ$S (through Andes mountain range) for different representations of surface elevation. The labeling is the same as in Figure 1.

The importance of anisotropy in quantifying topographic effects has been recognized for some time (e.g., Baines and Palmer, 1990; Bacmeister, 1993). According to linear theory gravity waves can propagate in the vertical only when their intrinsic frequency is lower than the Brunt-Väisälä frequency $N$ (e.g., Durran, 2003). For orographically-forced gravity waves the intrinsic frequency is set by the obstacle horizontal scale and the wind speed. When obstacle scales are too small to generate propagating waves we expect drag to produced by unstratified turbulent flow, a process which is typically parameterized in models’ TMS schemes. For larger obstacles we expect both drag and vertically-propagating waves to result, processes which are dealt by GWD schemes. Unfortunately the scale separating TMS and GWD processes is flow dependent. For typical midlatitude values of low-level wind (10 m s$^{-1}$) and $N$ (10$^{-2}$ s$^{-1}$) waves with wavelengths less than around 6000m will not propagate in the vertical. A separation scale of 5000 m has been used by ECMWF (1997); Beljaars et al. (2004).

Here we will generate two sub-grid-scale variables derived from the topography data: The variance of topography below the 6000m scale (referred to as $\text{Var}^{(\text{TMS})}$) and the variance of topography with a scale longer than 6000m and less than the grid scale (referred to as $\text{Var}^{(\text{GWD})}$).

It is the purpose of this paper to document a software package that, given a ‘raw’ high resolution global elevation dataset, maps elevations data to any unstructured global grid and consistently separates the scales needed for TMS and GWD parameterizations. This separation of scales is done through an intermediate mapping of the raw elevation data to a 3000 m cubed-sphere grid ($\Lambda \rightarrow A$) before mapping fields to the target model grid ($A \rightarrow \Omega$) as schematically shown on Figure 3. This two-step process is described in section 2.2 after a mathematical definition of the scale-separation (section 2.1). In section 2.2.3 we briefly discuss $\Phi_s$ smoothing. In the spirit of model development some exploratory experiments illustrating the impacts of topographic smoothing with the NCAR-DOE CAM-SE and, for comparison CAM-FV, are presented in section 3.2. Results from these experiments are shown to emphasize the importance of topographic smooth-
ing rather than detailed investigation of the accuracy and process level analysis of how to most accurately model flow over topography.

2 Method

2.1 Continuous: separation of scales

The separation of scales is, in continuous space, conveniently introduced using spherical harmonics. Assume that elevation (above sea level) is a smooth continuous function in which case it can be represented by a convergent expansion of spherical harmonic functions of the form

\[ h(\lambda, \theta) = \sum_{m=-\infty}^{\infty} \sum_{n=|n|}^{\infty} \psi_{m,n} Y_{m,n}(\lambda, \theta), \]

(1)

(e.g., Durran 2010) where \( \lambda \) and \( \theta \) are longitude and latitude, respectively, \( \psi_{m,n} \) are the spherical harmonic coefficients. Each spherical harmonic function is given in terms of the associated Legendre polynomial \( P_{m,n}(\theta) \):

\[ Y_{m,n} = P_{m,n}(\theta) e^{im\lambda} \]

(2)

where \( m \) is the zonal wave number and \( m - |n| \) is the number of zeros between the poles and can therefore be interpreted as meridional wave number.

For the separation of scales the spherical harmonic expansion is truncated at wave number \( M \)

\[ h^{(M)}(\lambda, \theta) = \sum_{m=-M}^{M} \sum_{n=|n|}^{M} \psi^{(M)}_{m,n} Y_{m,n}(\lambda, \theta), \]

(3)

where a triangular truncation, which provides a uniform spatial resolution over the entire sphere, is used.

Let \( h^{(tg)}(\lambda, \theta) \) be a continuous representation of the elevation containing the spatial scales of the target grid. We do not write \( h^{(tg)}(\lambda, \theta) \) explicitly in terms of spherical harmonics as the target grid may be variable resolution and therefore contains different spatial scales in different parts of the domain.

For each target grid cell \( \Omega_k \), \( k = 1, \ldots, N_t \), where \( N_t \) is the number of target grid cells, define the variances

\[ \text{Var}_{\Omega_k}^{(tm)} = \int_{\Omega_k} [h^{(M)}(\lambda, \theta) - h^{(M)}(\lambda, \theta)]^2 \cos(\theta) d\lambda d\theta, \]

\[ \text{Var}_{\Omega_k}^{(tg)} = \int_{\Omega_k} [h^{(tg)}(\lambda, \theta) - h^{(M)}(\lambda, \theta)]^2 \cos(\theta) d\lambda d\theta, \]

So \( \text{Var}_{\Omega_k}^{(tm)} \) is the variance of elevation on scales below wave number \( M \) and \( \text{Var}_{\Omega_k}^{(tg)} \) is the variance of elevation on scales larger than wave number \( M \) and below the target grid scale.

2.2 Discrete: separation of scales

The separation of scales is done through the use of a quasi-isotropic gnomonic cubed-sphere grid in a two-step regridding procedure: binning from source grid \( \Lambda \) to intermediate grid \( A \) (separation of scales) and then rigorously remap variables to the target grid \( \Omega \).

Any quasi-uniform spherical grid could, in theory, be used for the separation of scales. For reasons that will become clear we have chosen to use a gnomonic cubed-sphere grid (see Figure 3) resulting from an equi-angular gnomonic (central) projection

\[ x = r \tan \alpha \quad \text{and} \quad y = r \tan \beta; \quad \alpha, \beta \in \left[-\frac{\pi}{4}, \frac{\pi}{4}\right], \]

(6)

(Ronchi et al. 1996) where \( \alpha \) and \( \beta \) are central angles in each coordinate direction, \( r = R/\sqrt{3} \) and \( R \) is the radius of the Earth. A point on the sphere is identified with the three-element vector \((x, y, \nu)\), where \( \nu \) is the panel index. Hence the physical domain \( S \) (sphere) is represented by the gnomonic (central) projection of the cubed-sphere faces, \( \Omega^{(\nu)} = [-1, 1]^2 \). \( \nu = 1, 2, \ldots, 6 \), and the non-overlapping panel domains \( \Omega^{(\nu)} \) span the entire sphere: \( S = \bigcup_{\nu=1}^{6} \Omega^{(\nu)} \). The cube edges, however, are discontinuous. Note that any straight line on the gnomonic projection \((x, y, \nu)\) corresponds to a great-circle arc on the sphere. In the discretized scheme we let the number of cells along a coordinate axis be \( N_c \) so that the total number of cells in the global domain is \( 6 \times N_c^2 \). The grid lines are separated by the same angle \( \Delta \alpha = \Delta \beta = \frac{\pi}{N_c} \) in each coordinate direction.

For notational simplicity the cubed-sphere cells are identified with one index \( i \) and the relationship between \( i \) and \((icube, jcube, \nu)\) is given by

\[ i = icube + (jcube - 1) N_c + (\nu - 1) N_c^2, \]

(7)

where \((icube, jcube) \in [1, \ldots, N_c]^2 \) and \( \nu \in [1, 2, \ldots, 6] \). In terms of central angles \((\alpha, \beta)\) the cubed-sphere grid cell \( i \) is defined as

\[ \Lambda_i = [(icube - 1)\Delta \alpha - \pi/4, icube\Delta \alpha - \pi/4] \times [(jcube - 1)\Delta \beta - \pi/4, jcube\Delta \beta - \pi/4], \]

(8)

and \( \Delta A_i \) denotes the associated spherical area. A formula for the spherical area \( \Delta A_i \) of a grid cell on the gnomonic cubed-sphere grid can be found in Appendix C of Lauritzen and Nair (2008) (note that equation C3 is missing arccos on the right-hand side). A quasi-uniform approximately 3000 m resolution is obtained by using \( N_c = 3000 \) which results in a scale separation of roughly 6000 m. For more details on the construction of the gnomonic grid see, e.g., Lauritzen et al. (2010).

2.2.1 Step 1: raw elevation data to intermediate cubed-sphere grid (A \( \rightarrow \) A)

The ‘raw’ elevation data is usually from a digital elevation model (DEM) such as the GTOPO30; a 30 arc second global
Figure 3. A schematic showing the regridding procedure. Red fonts refer to the naming conventions for the grids: Λ is the ‘raw’ data grid (1km regular lat-lon grid), Α is the intermediate cubed-sphere grid and Ω is the target grid which may be any unstructured or structured grid. The variable naming conventions in the boxes are explained in section 2.3. Data for the variable resolution MPAS (Model for Prediction Across Scales; Skamarock et al., 2012) grid plot is courtesy of W. C. Skamarock (NCAR).
The land fraction is also binned to the intermediate cubed-sphere grid essentially an equidistant Cartesian grid on each panel in terms of the central angle coordinates. This step could be replaced by rigorous remapping in terms of overlap areas between the regular latitude-longitude grid and the cubed-sphere grid using the geometrically exact algorithm of [Ullrich et al. 2009] optimized for the regular latitude-longitude and gnomonic cubed-sphere grid pair or the more general remapping algorithm called SCRIP [Jones 1999].

2.2.2 Step 2: cubed-sphere grid to target grid ($A \rightarrow \Omega$)

The cell averaged values of elevation and sub-grid-scale variances ($V_{\Omega k}^{(tms)}$) and $V_{\Omega k}^{(gwd)}$ on the target grid are computed by rigorously remapping the variables from the cubed-sphere grid to the target grid. The remapping is performed using CSLAM (Conservative Semi-Lagrangian Multi-tracer transport scheme) technology [Lauritzen et al. 2010] that has the option for performing higher-order remapping. It is possible to use large parts of the CSLAM technology since the source grid is a gnomonic cubed-sphere grid hence instead of remapping between the gnomonic cubed-sphere grid and a deformed Lagrangian grid, as done in CSLAM transport, the remapping is from the gnomonic cubed-sphere grid to any target grid constructed from great-circle arcs (the target grid ‘plays the role’ of the Lagrangian grid). However, a couple of modifications where made to the CSLAM search algorithm. First of all, the target grid cells can have an arbitrary number of vertices whereas the CSLAM transport search algorithm assumes that the target grid consists of quadrilaterals and the number of overlap areas are determined by the deformation of the transporting velocity field. In the case of the remapping needed in this application the target grid consists of polygons with any number of vertices and the search is not constrained by the physical relation between regular and deformed upstream quadrilaterals. Secondly, the CSLAM search algorithm for transport assumes that the target grid cells are convex which is not necessarily the case for target grids. The CSLAM search algorithm has been modified to support non-convex cells that are, for example, encountered in variables resolution CAM-SE (see Figure 4), essentially that means that any target grid cell may cross a gnomonic isoline (source grid line) more than twice.

Let the target grid consist of $N_k$ grid cells $\Omega_k$, $k = 1, \ldots, N_t$ with associated spherical area $\Delta \Omega_k$. The search algorithm for CSLAM is used to identify overlap areas between the target grid cell $\Omega_k$ and the cubed-sphere grid cells $A_\ell$, $\ell = 1, \ldots, N_c$. Denote the overlap area between $\Omega_k$ and $A_\ell$:

$$\Omega_{k\ell} = \Omega_k \cap A_\ell,$$

(see Figure 5) and let $L_k$ denote the set of indices for which $\Omega_k \cap A_\ell$, $\ell = 1, \ldots, N_c$, is non-empty. Then the average eleva-
The appended superscript \textit{raw} in $\overline{h}^{(tgt,raw)}$ and $\overline{\var{\textit{raw}}}^{(tgt,raw)}$ refers to the fact that the elevation and GWD variance is based on unsmoothed elevation. As mentioned in the introduction the elevation is usually smoothed i.e. the highest wave number are removed. This smoothed elevation is denoted

$$\overline{h}^{(tgt,\text{smooth})},$$

(17)

Note that after smoothing the target grid elevation the sub-grid scale variance for GWD should be recomputed as the smoothing operation will add energy to the smallest wave lengths:

$$\overline{\var{\textit{raw}}}^{(tgt,\text{smooth})} = \frac{1}{\Delta\Omega_k} \sum_{\ell \in L_k} \left( \overline{h}^{(tgt,\text{smooth})}_{\Omega_k} - \overline{h}^{(\text{cube})}_{A_l} \right)^2 \Delta\Omega_{kl}.$$  

(18)

The smoothing of elevation is discussed in some detail in the next section.

### 2.2.3  Smoothing of elevation $h^{(tgt)}$

As discussed in the introduction the raw topographic data mapped to the target grid cell without further filtering to remove the highest wave numbers usually leads to excessive spurious noise in the simulations. There seem to be no standardized procedure, for example a test case suite, to objectively select the level of smoothing and filtering method. The amount of smoothing necessary to remove spurious noise in, e.g. vertical velocity, depends on the amount of inherent or explicit numerical diffusion in the dynamical core (e.g., Lauritzen et al., 2011). It may be considered important that the topographic smoothing is done with discrete operators consistent with the dynamical core.

While it is necessary to smooth topography to remove spurious grid-scale noise, it potentially introduces two problems. Filtering will typically raise ocean points near step topography to non-zero elevation. Perhaps the most striking example is the Andes mountain range that extends one or two grid cells into the Pacific after the filtering operation (Figure 2). Ocean and land points are treated separately in weather/climate models so raised sea-points may potentially be problematic. Secondly, the filtering will generally reduce the height of local topographic maxima and given the importance of barrier heights in atmospheric dynamics, this could be a problem for the global angular momentum budget and could fundamentally change the flow (unless a parameterization for blocking is used). To capture more of the barrier effect (blocking) , two approaches have been put forward in the literature to deepen valleys and increase peak heights while filtering out small scales: \textit{envelope topography} adjusts the surface height with sub-grid scale topographic variance (Wallace et al., 1983; Mesinger and Strickler, 1982). Loosely speaking, the otherwise smoothed peak heights are raised.

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**Figure 4.** An example of a non-convex control volume in variable resolution CAM-SE. Vertices are filled circles and they are connected with straight lines.

**Figure 5.** Schematic of the notation used to define the overlap between target grid cell $\Omega_k$ (red polygon) and high resolution cubed-sphere grid cell $A_l$ (Cartesian-like grid on Figure). The overlap area, $\Omega_{kl} = \Omega_k \cap A_l$, is shaded with cross-hatch pattern on Figure.
This process may perturb the average surface height. Alternatively, one may use Silhouette averaging that adds mass ... [Smart et al., 2005] Walko and Tremback [2004] Bossert [1990]. A similar approach, but implemented as variational filtering, is taken in Rutt et al. [2006]; this method also imposes additional constraints such as enforcing zero elevation over ocean masks.

As there is no standard procedure for smoothing topography, we leave it up to the user to smooth the raw topography \( h_{\text{tgt,raw}} \). The smoothed topography is referred to as \( h_{\text{tgt,smooth}} \). As mentioned before \( \Phi_{\text{tgt,smooth}} \) must be recomputed after smoothing \( \Phi \) as the filtering operation will transfer energy to the sub-grid-scale.

### 2.3 Naming conventions

The naming conventions for the topographic variables in the software and NetCDF files is:

\[
\begin{align*}
\text{PHIS} &= \bar{g} h_{\Omega}^{(\text{tgt})}, \\
\text{SGH} &= \sqrt{\text{Var}}_{\Omega}^{(\text{gw}d)}, \\
\text{SGH}30 &= \sqrt{\text{Var}}_{\Omega}^{(\text{tms})}, \\
\text{LANDFRAC} &= \overline{f}_{\Omega}^{(\text{tgt})},
\end{align*}
\]

where \( g \) is the gravitational constant.

### 3 Results

Exploratory simulations illustrating the effects of topographic smoothing on some climate diagnostics in CAM are shown.

#### 3.1 Smoothing methods used in CAM-FV and CAM-SE

In the CAM-FV the highest wave-numbers are removed by mapping \( \Phi_{s} \) (surface geopotential) to a regular latitude-longitude grid that is half the resolution of the desired model resolution, and then map back to the model grid by one-dimensional remaps along latitudes and longitudes, respectively, using the PPM (Piecewise Parabolic Method) with monotone filtering. In CAM-SE the surface geopotential is smoothed by multiple applications of the CAM-SE Laplace operator combined with a bounds preserving limiter. Figure 2 shows different levels of smoothing of surface height for CAM-SE and for comparison CAM-FV. It can clearly be seen that there are large differences between the height of the mountains with different smoothing operators and smoothing strength.

#### 3.2 Example topography smoothing experiments with CAM-SE

CAM-SE uses the spectral element dynamical core from HOMME (High-Order Method Modeling Environment;...
mapping of high resolution elevation data to an equi-angular cubed-sphere grid. The software supports structured or unstructured (e.g., variable resolution) global grid.

**Code availability**

The source codes for the NCAR Global Model Topography Generation Software for Unstructured Grid are available at through Github. The repository URL is https://github.com/UCAR/Topo/tags/. The repository also contains NCL scripts to plot the topography variables (Figure 6) that the software generates.

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**References**


Figure 7. Diagnostics for 30 year AMIP simulations with CAM5.2. Upper and lower group of plots are model level 16 vertical velocity and total precipitation rate differences, respectively. Except for the lower right-most plot on the lower group of plots, the diagnostics are for CAM-SE with different amounts of smoothing of $\Phi_s$ and different levels of divergence damping. The amount of smoothing follows the same notation as Fig. 2 (right) and $1.0 \times \text{div}$, $2.5 \times \text{div}$, $5.0 \times \text{div}$ refers to increasing divergence damping by a factor 1.0, $2.5^2$, and $5.0^2$, respectively. The second right-most plot on each group of plots (labeled FV) show results for CAM-FV. Lower right plot in the second group of plots show TRMM observations, respectively.

Figure 8. (left) Total kinetic energy spectrum for the velocity field at 200hPa as a function of spherical wave number $K$ for CAM-FV and different configurations of CAM-SE. The labeling for the CAM-SE configurations is the same as in Figure 7. The solid-straight black line indicates the $K^{-3}$ reference slope (Nastrom and Gage, 1985). The middle and right plots show the kinetic energy partitioned into divergent and rotational modes, respectively. The spectra have been computed using daily instantaneous wind and surface pressure data for a 2 month period.


