Timescales of mesoscale eddy equilibration in the Southern Ocean

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Southern Ocean Momentum Balance

Existing Theoretical Framework (Gill et. al. 1974, Marshall & Speer, 2012; Hallberg & Gnanadesikan, 2006; Abernathey & Cessi, 2014): Competition between wind driven upwelling and baroclinic eddies determines mean isopycnal slope, ACC transport, and MOC.
Meredith & Hogg (2006, GRL)

satellite altimetry

model

Meredith et al. (2012)
Power Spectra of Winds

NCAR Reanalysis

ERA Interim

(a) NCEP-NCAR

(b) ERA-Interim
EKE lags wind stress by 2 - 3 years
But why?

Wind Stress $\rightarrow$ APE $\rightarrow$ EKE

Simple model:
channel SO

Variable
Wind forcing at surface

Adiabatic Interior

wind power input
Conversion term
Drag

\[ \frac{d(APE)}{dt} = W - C \]
\[ \frac{d(EKE)}{dt} = C - D \]

\[ C = - \iiint \omega' b' dV \]
\[ \sim -K_{GM} \frac{(\nabla b)^2}{N^2} \]
Simple model: w/o Eddy feedback

\[ \frac{dK}{dt} = f(t) c_P(t) \]
\[ \frac{dP}{dt} = c_P(t) r_K(t) \]

Low Freq. Limit ("eddy saturation")
- SMALL change in isopycnal slope (APE)

High Freq. Limit (Ekman)
- BIG change in isopycnal slope (APE)
- SMALL change in EKE

Transfer Function
\[ \hat{K} = f, \quad \hat{P} = f \]

Phase Angle
\[ \phi_K - \phi_f, \quad \phi_P - \phi_f \]

\[ c \sim \frac{K_{GM}}{L_y^2} \approx 10^{-9} \text{ s}^{-1} (~3 \text{ years !}) \]
Simple model

w/o Eddy feedback

\[
\frac{dP(t)}{dt} = f(t) - cP(t)
\]

\[
\frac{dK(t)}{dt} = cP(t) - rK(t)
\]

Two conversion terms:

- \(c_1\), \(c_2\)

Same as \(c\), (eddy mixing coeff)

with Eddy feedback

Based on mixing length arguments

\[
\frac{dP}{dt} = f - kPK^\alpha
\]

\[
\frac{dK}{dt} = kPK^\alpha - r_1K^{\beta}
\]

general bottom drag

linearize and solve for

\[
\frac{dP'}{dt} = f' - c_1P' - c_2K'
\]

\[
\frac{dK'}{dt} = c_1P' + c_2K' - rK'
\]

Sinha & Abernathey (JPO, 2016)
Isopycnal GOLD model:
(Hallberg & Gnanadesikan, 2001, 2006; Howard et al. 2015)

- Reduced gravity model
- 4 km horizontal resolution
- Three isopycnal layers
- Wind forcing only

Eight experiments
- Steady sinusoidal wind jet (0.2 N/m^2)
- plus oscillations +/- (0.1 N/m^2), 0.25, 0.5, 1, 2, 4, 8, 16 year periods

Diagnostics
- EKE
- APE
- Wind Energy input

Sinha & Abernathey (JPO, 2016)
Spectral Analysis

energy input:
same power, different frequency

response:
different amplitudes

Sinha & Abernathey (JPO, 2016)
Composite Analysis

$N_T$ forcing cycles

wind forcing cycle

forcing period $T$

$M$ ensemble members

time series of each diagnostic for all ensemble members

averaged over all ensemble members and all forcing cycles

**composite** over all forcing cycles and all ensemble members (T periodic signal)
Composite Analysis

2880 day forcing

Eddy Kinetic Energy $[m^2 s^{-2}]$

Available Potential Energy $[m^2 s^{-2}]$

Wind Power $[W m^{-2}]$

Time [days]

Sinha & Abernathey (JPO, 2016)
\[
\begin{align*}
\frac{dP'}{dt} &= f' - c_1 P' - c_2 K' \\
\frac{dK'}{dt} &= c_1 P' + c_2 K' - r K'
\end{align*}
\]

**weak eddy feedback**

\[c_1 = \frac{f}{\bar{P}}; \quad c_2 = \frac{f}{2\bar{K}}\]

\[\sim 560 \text{ days} \quad \sim 157 \text{ days}\]

**strong eddy feedback**

\[c_1 = \frac{f}{\bar{P}}; \quad c_2 = \frac{f}{\bar{K}}\]

\[\sim 560 \text{ days} \quad \sim 78 \text{ days}\]

*Sinha & Abernathey (JPO, 2016)*
Discussions

- Eddy *generation* and *dissipation* - non-local in time

- Eddy *memory effect* - Time dependent *eddy* parameterization
Eddy Feedback vs Eddy Memory

Eddy Feedback
Southern Ocean
*(Sinha and Abernathey, 2016 JPO)*

Eddy Memory
Beaufort Gyre
*(Manucharyan et al. in press)*
Discussions

- Eddy *generation* and *dissipation* - non-local in time

- Eddy *memory effect* - Time dependent *eddy parameterization*

- Used in conjunction with multiple timescale response to *thermodynamic forcing* (Ferreira et al 2014) (sea ice, ozone depletion etc.) - more complete theory for SO response, *baroclinic eddy equilibration*
Summary

- Two limits: Fast vs Slow - Transient response to changing winds

- Analytical model: Energy Budget - wind power, APE, EKE
  - with and without eddy feedback

- Smooth transfer function, complex phase and amplitude response to changing winds: Regime shift

- Numerical simulations with idealized model

- Mechanistic description of the eddy equilibration process with purely dynamic forcing
Thank you.
Appendix
Simple model: Energy pathway

Wind work:
\[ \int \int \tau \cdot u \]
\[ W \]
\[ \rightarrow \]
\[ \int dA \]

Dissipation:
\[ D \]
\[ \rightarrow \]
\[ dA \cdot u \]

Steady-state balance:
\[ \frac{d(APE)}{dt} = W - C \]
\[ \frac{d(EKE)}{dt} = C - D \]

Wind power input:
\[ C = - \int \int \int dV \ \bar{w}' \bar{b}' \]
\[ \sim - K_{GM} \frac{(\nabla b)^2}{N^2} \]
Simple model: channel SO

Variable Wind forcing at surface

Adiabatic Interior

\[
C = - \int \int \int_{dV} \overline{w'b'}
\sim -K_{GM} \frac{(\nabla b)^2}{N^2}
\]

Simple model:
Energy pathway

\[
\frac{d(APE)}{dt} = W - C
\]

Drag term

\[
\frac{d(EKE)}{dt} = C - D
\]

wind power input

Conversion term
Simple model with Eddy Feedback

Gain: APE

Gain: EKE

Phase Angle: APE

Phase Angle: EKE

Legend:
- $r=0.25$, $c_2=0.25$
- $r=1$, $c_2=0.25$
- $r=4$, $c_2=0.25$
- $r=0.25$, $c_2=1$
- $r=1$, $c_2=1$
- $r=4$, $c_2=1$
- $r=0.25$, $c_2=4$
- $r=1$, $c_2=4$
- $r=4$, $c_2=4$
Spectral Amplitude Response

![Graph showing spectral amplitude response with lines for EKE, APE, and 1/ω against forcing period in years and gain from spectra on a logarithmic scale.]
Amplitude and Phase from Composite

Graph 1: Gain (from composite) vs. Forcing Period (years)

Graph 2: Phase shift vs. Forcing Period (years)