On the Potential Vorticity Dynamics of Eighteen Degree Water

Bruno Deremble
Florida State University

CLIMATE IMPLICATIONS OF FRONTAL SCALE AIR-SEA INTERACTION
(Boulder, CO, USA, August 5-7, 2013)
Subtropical mode water and SST fronts

- A thick density layer found equatorward of each WBC
- In the Atlantic Ocean $T \approx 18^\circ$
- relation to an SST front?

Hanawa and Talley, 2001
Potential Vorticity (PV) formalism

In our case, PV is a measure of the vertical thickness of an isopycnal layer (the more PV, the thinner is the layer).

Thomas (2005)
Potential Vorticity (PV) formalism

- In our case, PV is a measure of the vertical thickness of an isopycnal layer (the more PV, the thinner is the layer)

- PV can’t cross isopycnals

Thomas (2005)
Potential Vorticity (PV) formalism

- In our case, PV is a measure of the vertical thickness of an isopycnal layer (the more PV, the thinner is the layer).
- PV can’t cross isopycnals.
- PV extraction occurs at the outcrop/incrop and is measured by the vector $J$.

Thomas (2005)
Potential Vorticity (PV) formalism

- In our case, PV is a measure of the vertical thickness of an isopycnal layer (the more PV, the thinner is the layer).
- PV can't cross isopycnals.
- PV extraction occurs at the outcrop/incrop and is measured by the vector \( J \).

Thomas (2005)
Potential Vorticity (PV) formalism

In our case, PV is a measure of the vertical thickness of an isopycnal layer (the more PV, the thinner is the layer).

- PV can’t cross isopycnals
- PV extraction occurs at the outcrop/incrop and is measured by the vector $J$

Thomas (2005)
Potential Vorticity (PV) formalism

Thomas (2005)

- In our case, PV is a measure of the vertical thickness of an isopycnal layer (the more PV, the thinner is the layer).
- PV can't cross isopycnals.
- PV extraction occurs at the outcrop/incrop and is measured by the vector $J$.
We use an ocean model (NEMO in NATL12 configuration) and make use of the PV formalism to understand the mode water dynamics.
PV view of EDW
PV exchange at the surface — scaling laws

The PV loss (gain) at the surface is measured by the vertical component of the $\mathbf{J}$ vector. What is the leading mechanism for PV extraction in the Gulf Stream region?

\[ J_z = \rho Q_w + \omega_z \frac{D\sigma}{Dt} + (\mathbf{F} \times \nabla\sigma)_z \]  

(1)

\[ \mathbf{J} = 0 \]
PV exchange at the surface — scaling laws

The PV loss (gain) at the surface is measured by the vertical component of the \( \mathbf{J} \) vector. What is the leading mechanism for PV extraction in the Gulf Stream region?

\[
J_z = \rho Q_w + \omega_z \frac{D\sigma}{Dt} + (\mathbf{F} \times \nabla \sigma)_z = 0
\]

\[
J^B_z \sim -\frac{f \alpha Q_{net}}{h C_p}
\]
PV exchange at the surface — scaling laws

The PV loss (gain) at the surface is measured by the vertical component of the \( \vec{J} \) vector. What is the leading mechanism for PV extraction in the Gulf Stream region?

\[
\begin{align*}
J_z &= \rho Q_w + \omega_z \frac{D\sigma}{Dt} + (\vec{F} \times \nabla \sigma)_z \\
J_B^z &\sim - f \alpha Q_{net} h c_p \\
J_F^z &\sim \frac{\tau \times \nabla \sigma}{\rho \delta_e}
\end{align*}
\]
PV exchange at the surface — scaling laws

The PV loss (gain) at the surface is measured by the vertical component of the $J$ vector. What is the leading mechanism for PV extraction in the Gulf Stream region?

\[ J_z = \rho Q_w + \omega_z \frac{D\sigma}{Dt} + (F \times \nabla \sigma)_z \]  

- $J_z = 0$
- $J_z^{B} \approx -\frac{f\alpha Q_{\text{net}}}{hc_p}$
- $J_z^{F} \approx \frac{\tau \times \nabla \sigma}{\rho \delta_e}$

Using the exact formulation, we end up with a PV extraction of $0.008$ pvf ($\approx 1\%$ of the pv content)

= nothing!
Estimate of the interior flux using the Bernoulli function

Figure: Bernoulli function in the mode water area. max 1.3, min 0.3

→ Max PV extraction: $\Delta B \times \Delta \rho \times \Delta T = 1 \times 0.2 \times 0.25 = 0.05 \text{ pvf}$

in good agreement with the surface flux.
Numerical validation

- Mean PV in the control volume: 1.03 pvuye
- Variability: 0.01 pvuye
- Very stable

Figure: 10 years time series of the PV enclosed in the contour and in 26.15 – 26.35
Why so little variation?

- $Q \simeq (f + \nabla \times u) \frac{d\rho}{dz}$
- within a layer, the integrated pv is
  $$Q_i = \iiint (f + \nabla \times u) \frac{d\rho}{dz} dz dx dy$$
  $$Q_i = (f + \nabla \times u) \Delta \rho S$$
- the thickness does not matter

The variations on the previous curve are almost only due to the relative vorticity content.
Walin (1982) Formalism
Walin (1982) Formalism

\[ Q_{\text{net}} \]

\[ T_1 \quad T_2 \]

The water mass formation is given by the heat flux divergence.

The mode water is a consequence of this principle.
Walin (1982) Formalism

Mass and temperature (or density) budgets combined

\[ M \approx -\partial F/\partial \rho, \quad F \approx Q_{\text{net}} \]

The Water mass formation is given by the heat flux divergence.

The mode water is a consequence of this principle.
Walnin (1982) Formalism

- Mass and temperature (or density) budgets combined
- \( M \simeq -\frac{\partial F}{\partial \rho} \), with \( F \simeq Q_{\text{net}} \)

The Water mass formation is \( \sim \) given by the heat flux divergence

The mode water is a consequence of this principle.
Validation of Walin

dashed: \( \oint z_\rho u \), red: \( \int\int F \)
Mean \( \sim 2.5 \) Sv: 25% of the total volume renewed every year.
Walin reproduces the main variations
Conclusions

- Mode water is essentially the result of water mass formation via air–sea heat flux (linked to SST fronts).
- The global PV budget is negligible.

Mean eddy decomposition → \( z_\rho q u = \hat{q} z_\rho u + z_\rho u'' q'' \)

→ explicit role of the mean flow/eddies; link with the volume budget

See also

- Deremblé and Dewar (2013) *Volume and Potential Vorticity Budgets of Eighteen Degree Water*, jpo, accepted
- Deremblé and Dewar (2012) *First order scaling law for potential vorticity extraction due to wind*, jpo
Mean map

Figure: arrows 'normalized'
Figure: arrows 'normalized' — 'almost' downgradient field
Mean – Eddies decomposition

Classical decomposition:

\[ \bar{u} = \frac{1}{t_1 - t_0} \int_{t_0}^{t_1} u dt \]  \hspace{1cm} (2)

\[ u = \bar{u} + u' \]

Thickness averaging decomposition

\[ \hat{u} = \frac{\bar{z}_\rho u}{z_\rho} \]  \hspace{1cm} (3)

\[ u = \hat{u} + u' \]

\[ z_\rho q u = \hat{q} z_\rho u + z_\rho u'' q'' \]
Potential Vorticity dynamics

\[ \frac{\partial}{\partial t} \rho Q + \nabla \cdot \mathbf{J} = 0. \]  

\( \mathbf{J} \) is the sum of an advective and a non advective term:

\[ \mathbf{J} = \rho \mathbf{Q} \mathbf{u} + \mathbf{\omega} \frac{D \sigma}{Dt} + \mathbf{F} \times \nabla \sigma \]  

PV budget for the mode water. We end up with

\[ \oint \mathbf{J} d\mathbf{l}^\perp + \int \int J_z = 0 \]  

No diapycnal flux (impermeability theorem).

\[ \oint \mathbf{J} d\mathbf{l}^\perp = \hat{q} \oint \mathbf{u} z \rho d\mathbf{l}^\perp + \oint z \rho u'' q'' d\mathbf{l}^\perp + \oint \mathbf{N} d\mathbf{l}^\perp \]  

→ explicit role of the mean flow/eddies; link with the volume budget
Eddies and Mean flow

Mean:

$$\hat{q} \oint u z \rho d l \perp$$ \hspace{1cm} (8)

We already know $\oint u z \rho d l \perp$ from the volume budget. and $\hat{q}$ is constant (contour line).

→ PV extraction by mean flow: 0.25 pvu/ye

Eddies

$$\oint z \rho u'' q'' d l \perp$$ \hspace{1cm} (9)

→ PV input by eddies flow: 0.28 pvu/ye
Evaluation in the model

Figure: 10 years time series of the volume of water enclosed in the contour and in 26.15 – 26.35

- Mean volume of mode water: 11.5 Svye
- Variability: 1.5 Svye

On the Potential Vorticity Dynamics of Eighteen Degree Water