Comparison of Adaptive and Uniform 2D Galerkin Simulations

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Overview

- Motivation
- Results: what do we gain by using AMR?
- Next steps
NUMA
Non-hydrostatic Unified Model of the Atmosphere

- **dynamical core** inside the Navy's next generation weather prediction system NEPTUNE
- **unified across numerics** (contains Continuous and Discontinuous Galerkin methods)
- **unified across applications** (regional and global modeling)
- 3D, DG, MPI: **strong scaling** for explicit time integration (tested up to 32000 CPUs)
- 2D, serial: allows **dynamic AMR**
NUMA
Collaborators

Numerical Methods and Moisture
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Simone Marras, Applied Math, NPS

Physical Parameterization and Databases
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Saša Gaberšek, NRL-Monterey
Kevin Viner, NRL-Monterey
Alex Reinecke, NRL-Monterey
Eric Hendricks, NRL-Monterey

Time-Integrators and PETSc Interface
Emil Constantinescu, ANL
Debo Ghosh, ANL

Preconditioners and Iterative Solvers
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Les Carr, Applied Math, NPS

Riemann Solvers and Limiters
Dale Durran, University of Washington
Maria Lukacova, University of Mainz

Many-Core Implementation
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Lucas Wilcox, Applied Math, NPS
Tim Warburton, CAAM, Rice University
Dave Norton, NVIDIA
Daniel Abdi (soon to be) Applied Math, NPS

ESMF Interface
Tim Campbell, NRL-Stennis
Tim Whitcomb, NRL-Monterey

Data Structures Optimization
Michael Bader, Computer Science, TUM
Kaveh Rahnema, Computer Science, TUM
Alex Breuer, Computer Science, TUM

motivation
results
next steps
### Goal

<table>
<thead>
<tr>
<th>methods</th>
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</tr>
</thead>
<tbody>
<tr>
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### Goal

**methods**
- dynamic AMR, uniform meshes
- high order, low order

**applications**
- cloud simulations
- Hurricane simulations

For which of these applications should we use these methods and how should we use them?
Motivation
Warm air bubble test case with $\mu = 0.1m^2/s$
Motivation

Warm air bubble test case with $\mu = 0.1m^2/s$

L²-error: how much accuracy do we gain by using dynamic AMR?
Questions:

L²-error: how much accuracy do we gain by using dynamic AMR?

Results:
### Questions:

1. L²-error: how much accuracy do we gain by using dynamic AMR?

2. How does the benefit of AMR depend on the initial condition?

### Results:

Warm air bubble test case with $\mu = 0.1m^2/s$ at $t = 700s$
Questions:

1. $L^2$-error: how much accuracy do we gain by using dynamic AMR?

2. How does the benefit of AMR depend on the initial condition?

3. What is the benefit for the error of $\max(\theta)$?

Results:

Warm air bubble test case with $\mu = 0.1m^2/s$ at $t = 700s$
L²-error of uniform simulations as a function of number of floating point operations

Motivation

Results

Next steps
L$_2$-error of AMR simulations as a function of number of floating point operations

motivation

compare efficiency

results
L²-error of AMR simulations as a function of number of floating point operations

**motivation**

**compare efficiency**

**results**

- AMR
- uniform

2x faster
L^2-error of AMR simulations as a function of number of floating point operations

- AMR: 2x faster
- AMR: 2x more accurate

motivation
compare efficiency
results
L²-error of AMR simulations as a function of number of floating point operations

AMR reduces the L²-error by a factor of 2 for same amount of work.
### Questions:

1. **L²-error:** how much accuracy do we gain by using dynamic AMR?

2. How does the benefit of AMR depend on the initial condition?

3. What is the benefit for the error of $\max(\theta)$?

### Results:

AMR reduces the L²-error by a factor of 2 for same amount of work.

---

**Warm air bubble test case with $\mu = 0.1 \text{m}^2/\text{s}$ at $t = 700\text{s}$**
Three different initial profiles for $\theta'$ as a function of distance from the center of the bubble $r$

$$\theta'(r) = \theta_c \exp\left(-\frac{r}{a}^s\right)$$
Reference result for modified bubbles
resolution $\Delta x = 40$cm, time $t=700$s

motivation
results
next steps
L²-error for 3 different initial conditions
viscosity: $\mu = 0.1 \text{m}^2/\text{s}$

- AMR
- uniform

$s=2$
$s=4$
$s=6$

TFLOPs (total number of $10^{12}$ floating point operations)
L²-error for 3 different initial conditions
viscosity: $\mu=0.1\text{m}^2/\text{s}$

- AMR
- uniform

2x faster

2x more accurate

s=2
s=4
s=6
L^2-error for 3 different initial conditions
viscosity: $\mu=0.1\text{m}^2/\text{s}$

- AMR
- uniform

- 2x faster
- 2x more accurate
- 3x faster
- 3x more accurate

TFLOPs (total number of $10^{12}$ floating point operations)

motivation results next steps
For the three different initial conditions that we tested we get very similar benefit of AMR.

\( \text{L}^2\)-error for 3 different initial conditions

viscosity: \( \mu = 0.1 \text{m}^2/\text{s} \)
Questions for Today
Warm air bubble test case with $\mu = 0.1\text{m}^2/\text{s}$ at $t = 700\text{s}$

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motivation  results  next steps
absolute relative error of $\max(\theta)$
viscosity: $\mu = 0.1 m^2/s$

- $s=2$
- $s=4$
- $s=6$

motivation  results  next steps

TFLOPs (total number of $10^{12}$ floating point operations)
absolute relative error of $\max(\theta)$
viscosity: $\mu=0.1\text{m}^2/\text{s}$

- AMR
- uniform

5x faster
30x more accurate

$\ s=2\quad s=4\quad s=6\$
AMR reduces the error of $\max(\theta)$ by a factor around 30 for the same amount of work.
### Questions:

1. **L^2-error**: how much accuracy do we gain by using dynamic AMR?

2. How does the benefit of AMR depend on the initial condition?

3. What is the benefit for the error of \( \max(\theta) \)?

### Results:

- AMR reduces the L^2-error by a factor of 2 for the same amount of work.

- For the three different initial conditions that we tested, we get very similar benefit of AMR.

- AMR reduces the error of \( \max(\theta) \) by a factor around 30 for the same amount of work.
**Questions for Today**

Warm air bubble test case with $\mu = 0.1 \text{m}^2/\text{s}$ at $t = 700\text{s}$

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<td><strong>Next steps:</strong> 1. different refinement criteria (gradient, ...), 2. include moisture, 3. run comparison in 3D</td>
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Squall line simulation with NUMA
isosurface of cloud water content $q_c=0.0035$ at $t=7500s$
Squall line simulation with NUMA

isosurface of cloud water content $q_c=0.0035$ at $t=7500s$
Squall line simulation with NUMA
visualization with Maya® (see http://anmr.de for instructions)
Thank you for your attention!