A Hybridized Discontinuous Galerkin Method for Dynamic Cores of Atmospheric and Ocean General Circulation Models

Tan Bui-Thanh†*
† Department of Aerospace Engineering and Engineering Mechanics
* Institute for Computational Engineering and Sciences
The University of Texas at Austin, USA.

We present an efficient high order hybridized Discontinuous Galerkin method (HDG) for the dynamical cores of atmospheric and ocean general circulation models. In particular, we derive HDG schemes by hybridizing upwind DG methods, i.e., single-valued extra trace unknowns are introduced on the mesh skeleton so that the field unknowns on each mesh element can be computed independently as a function of the unknown traces in an element-by-element fashion. Extra equations are then introduced on the skeleton to enforce the uniqueness of numerical flux, and this gives rise to an algebraic system of equations to solve for the unknown traces.

In effect, the HDG framework reduces the spatial dimension by one, and hence it has much less, in fact minimal, number of coupled degree of freedoms, namely the unknown traces, to solve for as compared to standard DG methods. Once the traces are computed, the field unknowns can be recovered in parallel element-by-element independent of each other. As such, HDG method is intrinsically scalable and parallelizable, and hence well suited for current and future exascale supercomputer technologies with co-processors and accelerators.

We present numerical results on various PDEs including advection equation, compressible Euler and Navier-Stokes equations, and shallow water equations.