In this talk, I will briefly discuss a recent time-stepping algorithm for nonlinear PDEs that exhibit fast (highly oscillatory) time scales. PDEs of this form arise in many applications of interest, and in particular describe the dynamics of the ocean and atmosphere. The scheme combines asymptotic techniques (which are inexpensive but can have insufficient accuracy) with parallel-in-time methods (which, alone, can yield minimal speedup for equations that exhibit rapid temporal oscillations).

In order to implement this time-stepping method for general spatial domains (in 2D and 3D), a key component involves applying the exponential of skew-Hermitian operators. To this end, I will next present a new algorithm for doing so. This scheme has several advantages over standard methods, including the absence of any stability constraints in relation to the spatial discretization, and the ability to parallelize the computation in the time variable over as many characteristic wavelengths as resources permit (in addition to any spatial parallelization). I will also present examples on the linear 2D shallow water equations, where this method (in serial) is 1-2 orders of magnitude faster than both RK4 and the use of Chebyshev polynomials.