The High-order Adaptively Refined Dynamical Core (HARDCore) composes several compact numerical methods to easily facilitate intercomparison of atmospheric flow calculations on the sphere and in rectangular domains. This framework includes implementations of Spectral Elements, Discontinuous Galerkin, Flux Reconstruction, and Hybrid Finite Element methods with the goal of achieving optimal accuracy in the solution of atmospheric problems. Several advantages of this approach are discussed such as: improved pressure gradient calculation, numerical stability by vertical/horizontal splitting, optimal wave propagation by staggering of prognostic variables, and arbitrary order of accuracy. The local numerical discretization allows for high performance parallel computation and efficient inclusion of parameterizations. These techniques are used in conjunction with a non-conformal, locally refined, cubed-sphere grid for global simulations and standard Cartesian grids for simulations at the mesoscale. A complete implementation of the methods described is shown with verification test case simulation results based on standard literature.