Reconciling and improving formulations for thermodynamics and conservation principles in Earth System Models (ESMs)

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Reconciling and improving formulations for thermodynamics and conservation principles in Earth System Models (ESMs)

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Physics-Dynamics Coupling in Earth System Models (19w5153)

Organizers
Nicholas Kevlahan (McMaster University)
Peter Lauritzen (National Center for Atmospheric Research)

https://www.birs.ca/events/2019/5-day-workshops/19w5153
Outline (of manuscript)

Theoretical energetics/budgets (section 2)
Start with the dry hydrostatic primitive equations (HPE) and gradually increase the thermodynamic complexity by first adding water vapor and then condensates to the HPE. Special attention is given to the derivation of enthalpy terms (and associated reference states), latent heat terms and surface flux terms. For these models, a detailed explanation of the approximations made in large-scale models can be included rigorously. An in-depth discussion is included of surface fluxes and the complications arising due to falling precipitation and/or water entering the atmosphere using a single-component fluid approach.

Energy (existing & missing) budget terms of a climate model (section 3)
- See also Oksana Guba’s talk (for the purpose of this discussion E3SM and CAM are the same!)

Energy budget errors (section 4)
- Numerical truncation energy errors in dynamical cores (adiabatic). (see Lauritzen and Williamson, 2019)
- Physics–dynamics coupling errors due to spatial and temporal discretization errors. (see Donahue & Caldwell, 2020, Lauritzen and Williamson, 2019)
- Thermodynamic inconsistency energy errors in physics:
  ° As an illustration we discuss a specific example in some detail: coupling the CLUBB cloud parameterization package with the CAM climate model.
- Thermodynamic and vertical coordinate inconsistencies between dynamical core and parameterizations:
  * different vertical coordinates (see Lauritzen et al., in prep, for z-based MPAS coupling with p-based CAM)
  * different enthalpy definitions (e.g., FV3/SE coupled with CAM)
- Mass “clipping” errors and energy
Then it can be shown that the following globally integrated total energy equation holds:

\[
\begin{align*}
\frac{\partial}{\partial t} \iint \rho \left\{ K + \Phi_s + c_p \left( T + \sum_{\ell \in L_{H_2O}} m^{(\ell)} \left[ K + \Phi_s + c_p (T - T_{00}) + h^{(ice)} \right] \right. \\
+ m^{(wv)} L_{s,00} + m^{(liq)} L_{f,00} \right\} dA dz \\
= \iint \left\{ \sum_{\ell \in L_{H_2O}} F_{net}^{(\ell)} \left[ \tilde{K}_s + \Phi_s + c_p \left( \tilde{T}_s - T_{00} \right) + h^{(ice)} \right] + F_{net}^{(wv)} L_{s,00} + F_{net}^{(liq)} L_{f,00} + F_{net}^{(turb,rad)} \right\} dA.
\end{align*}
\]

(ice reference enthalpy, \( \tilde{T}_s \equiv T_{atm,s} = T_{surf,s} \))

Then it can be shown that the following globally integrated total energy equation holds:

\[
\begin{align*}
\frac{\partial}{\partial t} \iint \rho \left\{ K + \Phi_s + c_p \left( T + \sum_{\ell \in L_{H_2O}} m^{(\ell)} \left[ K + \Phi_s + c_p (T - T_{00}) + h^{(ice)} \right] \right. \\
+ m^{(wv)} L_{s,00} + m^{(liq)} L_{f,00} \right\} dA dz \\
= \iint \left\{ \sum_{\ell \in L_{H_2O}} F_{net}^{(\ell)} \left[ \tilde{K}_s + \Phi_s + c_p \left( \tilde{T}_s - T_{00} \right) + h^{(ice)} \right] + F_{net}^{(wv)} L_{s,00} + F_{net}^{(liq)} L_{f,00} + F_{net}^{(turb,rad)} \right\} dA.
\end{align*}
\]

(ice reference enthalpy, \( \tilde{T}_s \equiv T_{atm,s} = T_{surf,s} \))
Total energy equation

Assume:

- Primitive equations (hydrostatic, shallow atmosphere, ideal gas)
- Assume model top pressure is constant
- All components of moist air have the same temperature and move with the same horizontal velocity
- Assume that water entering the atmosphere (evaporation, snow drift, sea spray) has same temperature as water leaving the atmosphere (dew, liquid and frozen precipitation) Just for notational simplicity!

Then it can be shown that the following globally integrated total energy equation holds:

\[
\frac{\partial}{\partial t} \int \int \int \rho^{(d)} \left( \tilde{K} + \tilde{\Phi}_s + c_p^{(d)} \tilde{T} + \sum_{\ell \in \mathcal{L}_{H,O}} m^{(d)} \left[ \tilde{K} + \tilde{\Phi}_s + c_p^{(d)} (\tilde{T} - T_{00}) + h_{00}^{(ice)} \right] \\
+ \tilde{m}^{(wv)} L_{s,00} + \tilde{m}^{(liq)} L_{f,00} \right) dA dz \\
= \int \int \left\{ \sum_{\ell \in \mathcal{L}_{H,O}} F_{\text{net}}^{(d)} \left[ \tilde{K}_s + \tilde{\Phi}_s + c_p^{(d)} (\tilde{T}_s - T_{00}) + h_{00}^{(ice)} \right] + F_{\text{net}}^{(wv)} L_{s,00} + F_{\text{net}}^{(liq)} L_{f,00} + F_{\text{net}}^{(turb,rad)} \right\} dA.
\]

(94)

Now also assume that the energy equation is valid for grid mean values in the model (QUESTIONABLE ASSUMPTION! No sub-grid reservoir of energy …)
Total energy equation

Assume:
- Primitive equations (hydrostatic, shallow atmosphere, ideal gas)
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Then it can be shown that the following globally integrated total energy equation holds:

\[
\frac{\partial}{\partial t} \iiint \rho^{(d)} \left( \overline{K} + \overline{\Phi}_s + c_p^{(d)} T + \sum_{\ell \in \mathcal{L}_{H_2O}} m^{(\ell)} \left[ \overline{K} + \overline{\Phi}_s + c_p^{(\ell)} (T - T_{00}) + h_{00}^{(ice)} \right] \right)
\]

\[
\mathcal{L}_{H_2O} = \int' w \nu' + \overline{m^{(wv)}} L_{s,00} + \overline{m^{(liq)}} L_{f,00} \right) dA dz
\]

\[
= \iint \left\{ \sum_{\ell \in \mathcal{L}_{H_2O}} \overline{F_{net}^{(\ell)}} \left[ \overline{\Phi}_s + c_p^{(\ell)} (\overline{\Theta} - T_{00}) + h_{00}^{(ice)} \right] + \overline{F_{net}^{(wv)}} L_{s,00} + \overline{F_{net}^{(liq)}} L_{f,00} + \overline{F_{net}^{(turb,rad)}} \right\} dA.
\]

(ice reference enthalpy, \( \overline{T}_s \approx \overline{T_{atm,s}} = \overline{T_{surf,s}} \))

Lauritzen et al. (2022, submitted)

Many models make these assumptions:

E.g. FV3 and NCAR-SE use variable latent heats and CAM physics not -> leads to 0.5W/m² imbalance (Lauritzen and Williamson, 2019)

Now also assume that the energy equation is valid for grid mean values in the model (QUESTIONABLE ASSUMPTION! No sub-grid reservoir of energy …)
Assume:
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Then it can be shown that the following globally integrated total energy equation holds:

$$\frac{\partial}{\partial t} \iint \rho(d) \left( K + \bar{\Phi}_s + c_p(d)T \right) dz$$

Many models make these assumptions:

Now also assume that the energy equation is valid for grid mean values in the model (QUESTIONABLE ASSUMPTION! No sub-grid reservoir of energy …)
Total energy equation

Assume:
- Primitive equations (hydrostatic, shallow atmosphere, ideal gas)
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Then it can be shown that the following globally integrated total energy equation holds:

$$\frac{\partial}{\partial t} \iiint \rho^{(d)} \left( K + \Phi_s + c_p^{(d)}T + \sum_{\ell \in \mathcal{L}_{H_2O}} m^{(\ell)} \left( K + \Phi_s + c_p^{(\ell)} (T - T_00) + h^{(ice)} \right) \right) + m^{(wv)} L_{s,00} + m^{(liq)} L_{f,00} \right) dA \, dz \right)$$

Many models make these assumptions:

$$\mathcal{L}_{H_2O} = \int \{ \sum_{\ell \in \mathcal{L}_{H_2O}} \left[ F_{net}^{(d)} \left( K + \Phi_s + c_p^{(d)} (T_s - T_00) + h^{(ice)} \right) + F_{net}^{(wv)} L_{s,00} + F_{net}^{(liq)} L_{f,00} + F_{net}^{(turb,rad)} \right] \} dA.$$ (94)

Now also assume that the energy equation is valid for grid mean values in the model (QUESTIONABLE ASSUMPTION! No sub-grid reservoir of energy …)

Lauritzen et al. (2022, submitted)
Updating water (pressure) in physics

\[ \Delta T^{(CAM)}_{\partial m^{(w)} / \partial t} = \Delta \hat{T}^{(CAM)}_{\partial m^{(w)} / \partial t} + \Delta \hat{T}^{(CAM)}_{\partial m^{(w)} / \partial t} \]

\[ = \int \frac{\partial}{\partial t} \left[ \rho^{(d)} \left(1 + \bar{m}^{(w)}\right) \right] \left( \Phi_s + c_p^{(d)}T \right) \, dz \]
From energy perspective it is problematic to consistently represent rain from the point at which it becomes falling precipitation: frictional dissipation (Pauluis et al., 2000), $T_s$, drag exerted by rain. **Note:** possible to consistently incl. frictional dissipation of rain by using barycentric velocity framework (see Appendix F in Lauritzen et al., 2022, submitted).

Modified CAM total energy equation incl. missing flux terms

$$\frac{\partial}{\partial t} \int \rho^{(d)} \left\{ \left( 1 + \bar{m}^{(H_2O)} \right) \left[ \bar{K} + \bar{\Phi}_s + c_p^{(d)} (T - T_{00}) \right] + \bar{m}^{(wv)} L_{s,00} + \bar{m}^{(liq)} L_{f,00} \right\} dz$$

$$- \Delta \bar{T} \frac{\partial m^{(H_2O)}}{\partial t} = \bar{F}^{(H_2O)} \left[ c_p^{(d)} \left( \bar{T}_s - T_{00} \right) + \bar{K}_s + \bar{\Phi}_s \right] + \bar{F}^{(wv)} L_{s,00} + \bar{F}^{(liq)} L_{f,00} + \bar{F}^{(turb,rad)}$$

**WARNING:** depends on reference state
Modified (consistent) total energy equation assuming variable latent heats

\[
\frac{\partial}{\partial t} \int \rho^{(d)} \left\{ \left( 1 + \bar{m}^{(H_2O)} \right) (\bar{K} + \Phi_s) + c_p^{(d)} T + \sum_{\ell \in L_{H_2O}} \bar{m}^{(\ell)} c_p^{(\ell)} (\bar{T} - T_{00}) + \bar{m}^{(liq)} L_{s,00} + \bar{m}^{(liq)} L_{f,00} \right\} dz \\
- \Delta \ell L(T) - \Delta \ell L(T) = - \sum_{\ell \in L_{H_2O}} F^{(\ell)}_{\text{net}} \left[ c_p^{(\ell)} \left( \bar{T}_s - T_{00} \right) + \bar{K}_s \right] + F^{(uv)}_{\text{net}} L_{s,00} + F^{(liq)}_{\text{net}} L_{f,00} + F^{(turb,rad)}_{\text{net}}
\]

Lauritzen et al. (2022, submitted)

(a) Imbalance for processes not involving falling precip. & evap.

(b) Imbalance for falling precip. & evap.
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Summary and future directions
Thermodynamic conserved variable inconsistency leading to total energy errors

An example: Coupling CLUBB with CAM (problem identified by Chris Golaz in 2010)

In detail, CLUBB transports an approximate form of the conserved moist potential temperature $\theta_l$ (see Tripoli & Cotton, 1981; Cotton et al., 2011), which is defined as

$$\theta_l \equiv T \Pi^{-1} - \frac{L_v,00}{c_p^{(d)}} \Pi^{-1} m^{(liq)},$$  

(151)

where $\Pi$ is the Exner function, which is purely a function of pressure. CLUBB then returns to CAM the following tendency of $\theta_l$,

$$\overline{\rho^{(d)}} (1 + \overline{m^{(wv)}}) \frac{\partial \theta_l}{\partial t} \bigg|_{\text{CLUBB}} = - \frac{\partial}{\partial z} \left[ \overline{\rho^{(d)}} (1 + \overline{m^{(wv)}}) \overline{w'} \theta'_l \right],$$  

(152)

(152) in terms of T

Assuming no surface fluxes and K changes in CLUBB

$$\int \frac{1}{\Pi t_n} (c_p^{(d)} \Delta T - L_v,00 \Delta m^{(liq)}) \overline{\rho^{(d)}} (1 + \overline{m^{(wv)}}) \overline{w'} t_n \overline{\Pi} d_z = 0.$$  

(155)

Lauritzen et al. (2022, submitted)
Thermodynamic conserved variable inconsistency leading to total energy errors

An example: Coupling CLUBB with CAM (problem identified by Chris Golaz in 2010)

\[
\int \left( c_p^{(d)} \Delta T - L_{v,00} \Delta m^{(liq)} \right) \frac{\rho^{(d)}}{\Pi^{tn}} (1 + m^{(wv)}_{tn}) \, dz = 0. \tag{158}
\]

CAM’s conserved variable

\[
\int \frac{1}{\Pi^{tn}} \left( c_p^{(d)} \Delta T - L_{v,00} \Delta m^{(liq)} \right) \frac{\rho^{(d)}}{\Pi^{tn}} (1 + m^{(wv)}_{tn}) \, dz = 0. \tag{155}
\]

CLUBB’s conserved variable

Lauritzen et al. (2022, submitted)
1-year column averaged imbalance using CAM (CESM)

Lauritzen et al. (2022, submitted)
Thermodynamic inconsistency in sensible heat flux in CAM-CLUBB

Neglect kinetic energy (i.e. assume for the moment that CLUBB does not alter winds), neglect radiation and assume that there are no phase changes. Then CAM’s energy equation reduces to:

$$\frac{\partial}{\partial t} \int \left\{ \bar{\rho}^{(d)} \left[ 1 + \bar{m}^{(wv)}_{t=t^n} \right] c_p^{(d)} \bar{T} \right\} dz = \bar{F}_{net}^{(turb)}.$$  \hspace{1cm} (159)

In contrast, CLUBB conserves

$$\frac{\partial}{\partial t} \int \left\{ \bar{\rho}^{(d)} \left[ 1 + \bar{m}^{(wv)}_{t=t^n} \right] \theta' \right\} dz = \bar{\rho}^{(d)} \left[ 1 + \bar{m}^{(wv)}_{t=t^n} \right] \left. w' \theta'_l \right|_{surface} \equiv \bar{F}_{net}^{(turb)}.$$  \hspace{1cm} (160)

That is, CLUBB conserves a potential temperature variable rather than temperature. In the absence of phase changes, (160) becomes

$$\frac{\partial}{\partial t} \int \left\{ \bar{\rho}^{(d)} \left[ 1 + \bar{m}^{(wv)}_{t=t^n} \right] \frac{T}{\Pi} \right\} dz = \bar{F}_{net}^{(turb)}.$$  \hspace{1cm} (161)

Sensible heat flux should be scaled with Exner - currently not done in CAM (changing soon though!)
Thermodynamic inconsistency in sensible heat flux in CAM-CLUBB

CLUBB sensible heat flux consistency experiments

Lauritzen et al. (2022, submitted)