PARAMETERIZING EDDIES IN OCEAN CLIMATE MODELS

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Abstract

Gent and McWilliams (1990) proposed a parameterization for the effects of mesoscale eddies on the large-scale flow for use in ocean climate models. It proposes that eddies advect tracers in addition to mixing them along isopycnal surfaces. This parameterization can be considered as a generalization of the residual-mean meridional circulation of Andrews and McIntyre (1976) to three dimensions. The resulting Eliassen-Palm fluxes have to be parameterized in order to determine the momentum equation used in non-eddy-resolving ocean climate models. The Antarctic Circumpolar Current region of the ocean is most like the atmosphere in that there are no continents to block the zonal jet. The nonacceleration theorem would suggest that the eddy-induced advection of tracers should oppose mean flow advection at the latitude of Drake Passage, and this is confirmed in global ocean model simulations.

1. Introduction

The ocean component of coupled climate models must be integrated for hundreds to thousands of years in both uncoupled and coupled modes. This has meant that these components have used non-eddy-resolving resolution, although some are now starting to enter the eddy-permitting regime, with a horizontal resolution of about 1°. However, in both these regimes it is very important to have a good parameterization of the effects of the unresolved mesoscale ocean eddies on the large-scale flow.

This problem will be related to the residual-mean meridional circulation of Andrews and McIntyre (1976), and to a generalization of Eliassen-Palm fluxes. These concepts were developed to analyse aspects of stratospheric flow, and are elegantly discussed by Andrews *et al.* (1987) in sections 3.5, 3.6 and elsewhere in that textbook. The material in this article comes mostly from the paper by Gent and McWilliams (1996), and the figures are taken from the paper by Danabasoglu and McWilliams (1995).

2. Generalized Eliassen-Palm Fluxes

The governing equations for incompressible, Boussinesq, hydrostatic, adiabatic flow in cartesian coordinates are

$$\frac{D}{Dt}\mathbf{u} + f\mathbf{k} \times \mathbf{u} + \nabla p = 0, \tag{1}$$

$$\frac{D}{Dt}\rho = 0, \tag{2}$$

$$p_z + g\rho/\rho_0 = 0, \tag{3}$$

$$\nabla \cdot \mathbf{u} + w_z = 0, \tag{4}$$

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla + w\frac{\partial}{\partial z}. \tag{5}$$

$(\mathbf{u}, w)$ is the three-dimensional velocity, ∇ is the horizontal gradient operator, ρ is the density, and p is the pressure divided by a reference density ρ_0. The variables are now decomposed into low-pass components, denoted by an overbar, and eddy components by a low-pass filtering operator in time and space. This operator is difficult to define explicitly, but it cannot be either a time-average or a zonal-average. The reason is that non-eddy-resolving ocean models have both time and longitudinal variability in their solutions. Now define the filtered residual, denoted by a hat, and modified velocities by

$$\widehat{ab} = \overline{ab} - \overline{a}\overline{b}, \tag{6}$$

$$\mathbf{U} = \overline{\mathbf{u}} - (\widehat{\mathbf{u}\rho}/\overline{\rho}_z)_z, \tag{7}$$

$$W = \overline{w} + \nabla \cdot (\widehat{\mathbf{u}\rho}/\overline{\rho}_z). \tag{8}$$

The velocities in equations (7) and (8) generalize the usual definition of the zonally-averaged residual-mean circulation to three dimensions by including a zonal component. The modified velocity still obeys the usual continuity equation, and a modified substantial derivative can be defined that advects with $(\mathbf{U}, W)$. Thus,

$$\nabla \cdot \mathbf{U} + W_z = 0, \tag{9}$$

$$\frac{D^*}{Dt} = \frac{\partial}{\partial t} + \mathbf{U} \cdot \nabla + W \frac{\partial}{\partial z}. \tag{10}$$

The filtered equations from (1) - (3) can then be written exactly in the form

$$\frac{D^*}{Dt}\overline{u} - fV + \overline{p}_x = \nabla^{3D} \cdot \mathbf{E}, \tag{11}$$

$$\frac{D^*}{Dt}\overline{v} + fU + \overline{p}_y = \nabla^{3D} \cdot \mathbf{F}, \tag{12}$$

$$\frac{D^*}{Dt}\overline{\rho} = -G_z, \tag{13}$$

$$\overline{p}_z + g\overline{\rho}/\rho_0 = 0, \tag{14}$$

where ∇^{3D} is the three-dimensional gradient operator. Equations (11) - (14) extend the usual definition of the transformed Eulerian-mean equations, and $(\mathbf{E}, \mathbf{F}, G)$ are generalized Eliassen-Palm fluxes defined by

$$\begin{aligned} \mathbf{E} = \Big[& \overline{u}_z \widehat{u\rho}/\overline{\rho}_z - \widehat{uu}, \quad \overline{u}_z \widehat{v\rho}/\overline{\rho}_z - \widehat{vu}, \\ & - \overline{u}_x \widehat{u\rho}/\overline{\rho}_z + (f - \overline{u}_y)\widehat{v\rho}/\overline{\rho}_z - \widehat{wu} \Big], \end{aligned} \tag{15}$$

$$\begin{aligned} \mathbf{F} = \Big[& \overline{v}_z \widehat{u\rho}/\overline{\rho}_z - \widehat{uv}, \quad \overline{v}_z \widehat{v\rho}/\overline{\rho}_z - \widehat{vv}, \\ & - (f + \overline{v}_x)\widehat{u\rho}/\overline{\rho}_z - \overline{v}_y \widehat{v\rho}/\overline{\rho}_z - \widehat{wv} \Big], \end{aligned} \tag{16}$$

$$G = \overline{\rho}_x \widehat{u\rho}/\overline{\rho}_z + \overline{\rho}_y \widehat{v\rho}/\overline{\rho}_z + \widehat{w\rho}. \tag{17}$$

It is clear that if the filtered equations are to retain the adiabatic assumption of the original equation (2), then G_z in (13) must be zero. This is true if mesoscale eddy density fluxes are assumed to be aligned along mean isopycnals, which are surfaces of constant density. Making G equal zero could also be used as a definition of the low-pass filtering operator in time and space. For either reason, G is now set to zero, so that the filtered equations are also adiabatic.

3. Parameterizing the Eddy Density Fluxes

It is well known that baroclinic instability feeds off the potential energy of the mean flow and transfers energy to the mesoscale eddies. If an eddy-resolving ocean model is run adiabatically with only wind forcing, then there will be a domain-averaged transfer of mean to eddy potential energy by baroclinic instability. This will be balanced by a net conversion of mean kinetic to mean potential energy, even though the model is adiabatic. In a non-eddy-resolving model, this can be parameterized as follows. The equation for potential energy can be written in the form

$$\frac{D^*}{Dt}\left(g\overline{\rho}z\right) = g\overline{\rho w} + \nabla \cdot \left(g\overline{\rho}\ \widehat{\mathbf{u}\rho}/\overline{\rho}_z\right) - g\nabla\overline{\rho} \cdot \widehat{\mathbf{u}\rho}/\overline{\rho}_z. \tag{18}$$

The first term on the right-hand-side of equation (18) is the transfer term from mean kinetic energy. The second term integrates to zero in domain-average with suitable boundary conditions. The third term shows that a domain-averaged sink of potential energy can be assured if a simple downgradient form is assumed for the eddy density fluxes

$$\widehat{\mathbf{u}\rho} = -\kappa\nabla\overline{\rho}, \tag{19}$$

where κ is positive everywhere. Note that $\overline{\rho}_z$ is negative in statically stable flow. This choice was made in Gent and McWilliams (1990), but is justified and explained much more clearly in Gent *et al.* (1995).

4. Parameterizing the Eliassen-Palm Fluxes

One choice for the Eliassen-Palm fluxes was proposed in Gent and McWilliams (1996). The choice is

$$\mathbf{E} = \Big[\nu_H(\overline{u}_x - \overline{v}_y), \quad \nu_H(\overline{u}_y + \overline{v}_x), \quad \nu_V\overline{u}_z + f\widehat{v\rho}/\overline{\rho}_z\Big], \tag{20}$$

$$\mathbf{F} = \Big[\nu_H(\overline{v}_x + \overline{u}_y), \quad \nu_H(\overline{v}_y - \overline{u}_x), \quad \nu_V\overline{v}_z - f\widehat{u\rho}/\overline{\rho}_z\Big]. \tag{21}$$

The Eliassen-Palm flux divergences are then almost downgradient horizontal and vertical momentum diffusion, plus the appropriate Coriolis term. The resulting parameterized non-eddy-resolving momentum equation is

$$\frac{D^*}{Dt}\overline{\mathbf{u}} + f\mathbf{k}\times\overline{\mathbf{u}} + \nabla\overline{p} = \nabla\cdot(\nu_H\nabla\overline{\mathbf{u}}) + \mathrm{J}_{xy}(\nu_H, \mathbf{k}\times\overline{\mathbf{u}}) + (\nu_V\overline{\mathbf{u}}_z)_z, \tag{22}$$

where J is the two-dimensional Jacobian operator. Note that this Jacobian term is nonzero only when the horizontal viscosity has spatial dependence, and is necessary to ensure that no stress results from a uniform rotation.

The kinetic energy equation based on the momentum equation (22) is

$$\begin{aligned}&\frac{D^*}{Dt}\left(\frac{\overline{\mathbf{u}}\cdot\overline{\mathbf{u}}}{2}\right)+\nabla^{3D}\cdot\mathbf{H}+g\overline{\rho w}/\rho_0=\\&-\nu_H[(\overline{v}_x+\overline{u}_y)^2+(\overline{u}_x-\overline{v}_y)^2]-\nu_V\overline{\mathbf{u}}_z\cdot\overline{\mathbf{u}}_z.\end{aligned}\tag{23}$$

The normal component of the vector $\mathbf{H}$ is zero at the boundaries, so that this term integrates to zero in domain-average. The last term on the left-hand-side of (23) is the transfer to potential energy term, and the right-hand-side assures a domain-average sink of kinetic energy.

There is still much debate about, and no concensus on, the form of the momentum equation that should be used in non-eddy-resolving ocean models. Equation (22) has not been used in models so far; every ocean model uses the usual substantial derivative, defined by equation (5), and not the modified substantial derivative, defined by equation (10).

5. The Nonacceleration Theorem

The nonacceleration theorem was first described by Andrews and McIntyre (1976), although there is both earlier and later work, see section 3.6 of Andrews *et al.* (1987). The theorem was developed and applied mostly to flow in the stratosphere, which is often nearly adiabatic. First, the filtering must be zonal averaging which eliminates the pressure gradient term in equation (11). Andrews and McIntyre then proved that if the eddies are steady, linear waves then the divergences of the Eliassen-Palm fluxes in equations (11) and (13) are zero. Then there exists a steady state solution with $V = W = 0$. In this case, the eddy-induced velocity is equal and opposite to the mean velocity, and their advective effects on passive tracers, for example, cancel.

The only place in the world's ocean where the nonacceleration theorem is relevant is at the latitude of Drake Passage, where no continent blocks the zonal jet called the Antarctic Circumpolar Current (ACC). There are pressure differences across an enclosed ocean basin, so that the zonally-integrated pressure gradient term in (11) is nonzero. Recall that the filtering operator in this work is not zonal averaging, and that ocean eddies are not steady, linear waves. However, the nonacceleration theorem strongly suggests that the effects of the eddy-induced velocity should counteract those of the mean flow in the ACC. This is precisely what occurs in coarse resolution numerical models of the world ocean.

6. Global Ocean Model Results

These results come from a very early, coarse resolution version of the NCAR CSM Ocean Model (NCOM). CSM is the Climate System Model, and the ocean component was developed from the Modular Ocean Model (MOM) that was developed and maintained at the Geophysical Fluid Dynamics Laboratory. Details about this particular model setup can be found in Danabasoglu and McWilliams (1995). This version has a zonal resolution of 4°, a meridional resolution of 3°, and 20 levels in the vertical varying from 50m near the surface to 450m in the deep ocean. The prognostic model variables are horizontal velocity, potential temperature and salinity, with the density calculated from the equation of state for seawater. The eddy effects on potential temperature and salinity are parameterized as eddy-induced advection, as in equation (13), plus mixing terms oriented along and perpendicular to the mean isopycnals. Using the approximation of small isopycnal slopes gives the equation for potential temperature, T, in the form

$$\frac{D^*}{Dt}T = Q + R(\kappa_I, T) + (\kappa_V T_z)_z, \tag{24}$$

where Q is the surface heat flux and R represents Laplacian mixing along isopycnals.

The results are from an equilibrium solution that has been integrated for several thousand years. Figure 1 shows the annual-mean, zonally integrated meridional overturning streamfunction in Sverdrups (Sv) from the mean velocity and from the eddy-induced velocity. The only region of the world ocean where the eddy-induced velocity transports a significant amount is in the region of the ACC between 45°S and 60°S where the transport is more than 25 Sv near the surface. Note also that, as anticipated from the nonacceleration theorem, the eddy-induced streamfunction is the same magnitude as that from the mean velocity and opposite in sign. This means that the meridional streamfunction from the residual-mean velocities (V, W) will be small in the ACC. This is confirmed in figure 2b which shows the annual-mean, zonally averaged meridional overturning streamfunction calculated from (V, W). Figure 2a shows the meridional overturning streamfunction from

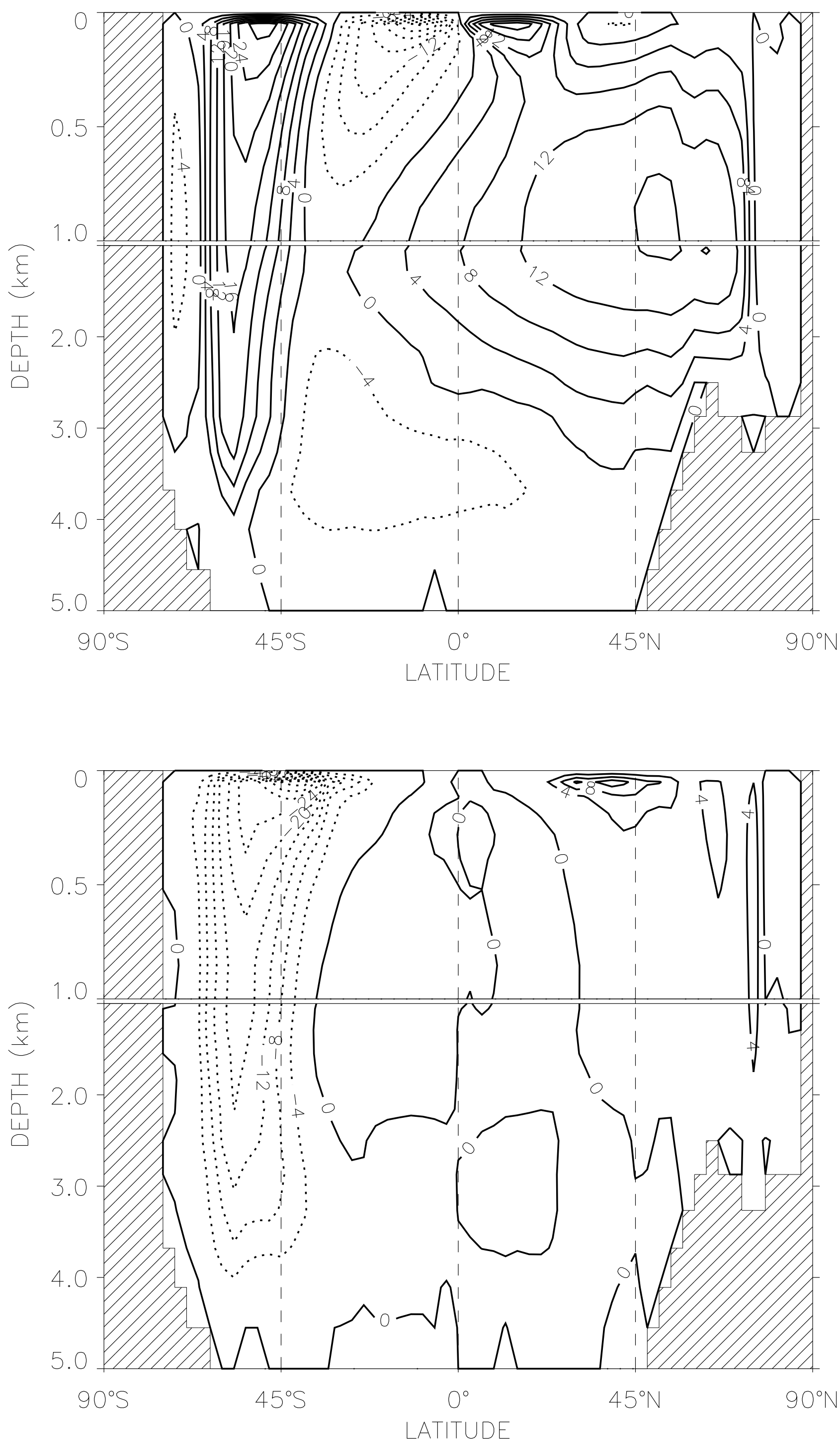

Figure 1. The annual-mean, zonally-averaged meridional overturning streamfunction in Sv from a) the mean velocity, and b) the eddy-induced velocity from an equilibrium solution using equation (24).

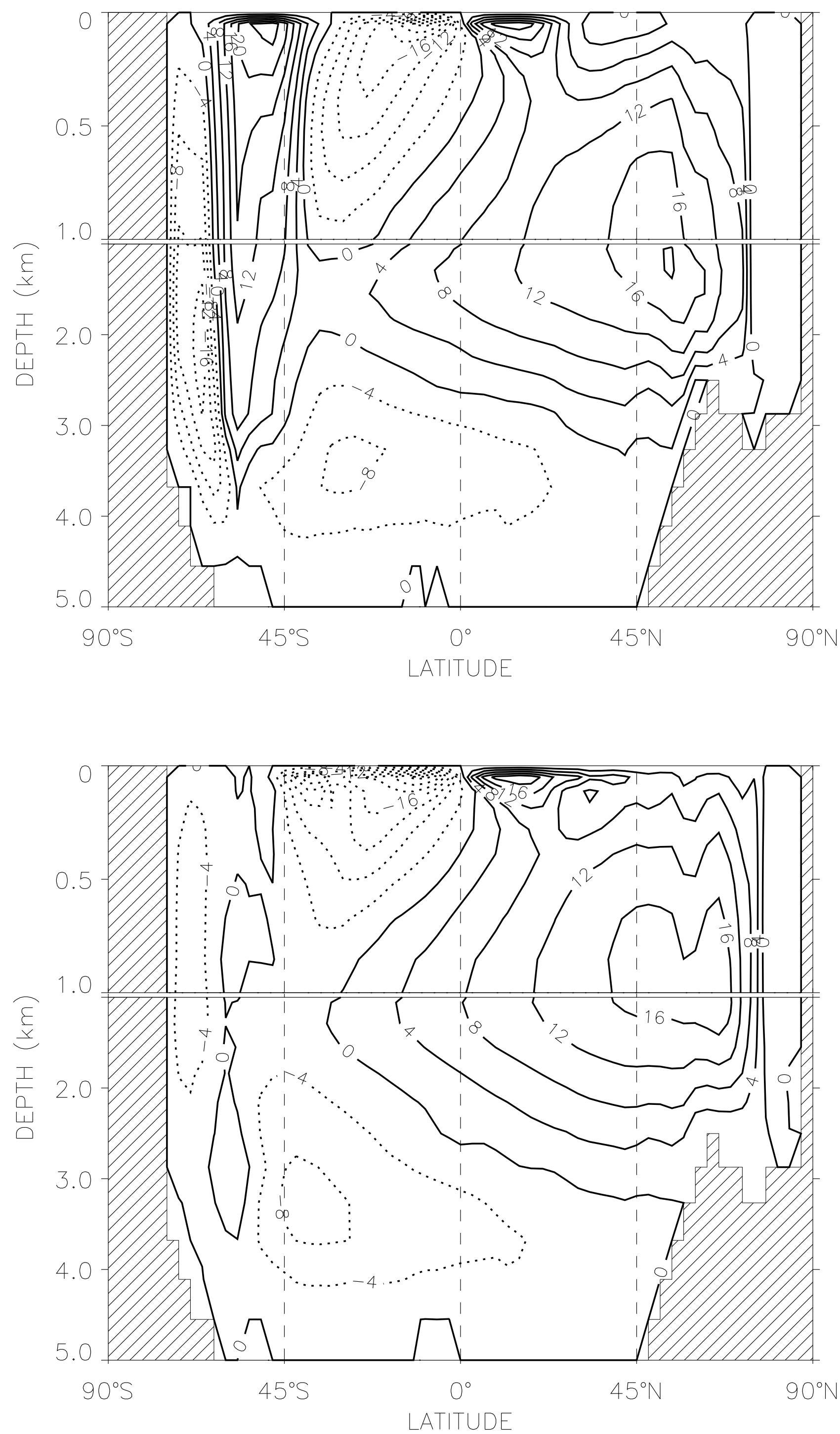

Figure 2. The annual-mean, zonally-averaged meridional overturning streamfunction in Sv from a) the mean velocity using horizontal tracer mixing, and b) the residual-mean velocity using equation (24).

a companion calculation using the earlier eddy parameterization that mixes T and S horizontally and has no additional advection. Figure 2a shows a strong cell in the ACC, similar to that in figure 1a.

The obvious conclusion from figure 2 is that in the ACC region, T and S will be advected very differently in the two companion calculations using horizontal tracer mixing and the tracer equation (24). One of the most important consequences of this different advection is the extent of convective adjustment in the ACC region in the two calculations. In a model using the hydrostatic approximation (14), vertical transfer of water is often accomplished by convective adjustment which is activated when the water column becomes statically unstable. In high latitudes, this can occur over most of the water column, and this is the model representation of deep water formation. Therefore, a measure of the location and frequency of deep water formation is easily diagnosed as when convective adjustment is triggered. This is shown in figure 3a,b from the calculations using horizontal tracer mixing and equation (24). In the first case, then is very extensive deep water formation over much of the ACC region, as well as in the North Atlantic and Arctic Oceans. In stark contrast, figure 3b shows deep water formation in rather few places, and is very much more realistic. Deep water formation is restricted to the Weddell and Ross Seas in the southern hemisphere and to the high North Atlantic and Arctic Oceans in the northern hemisphere. This represents a dramatic improvement in the ability of coarse resolution ocean models to simulate the global thermohaline circulation, which is very important for climate studies.

There are other improvements in results using equation (24) instead of horizontal tracer mixing. The deep water masses are much better represented because their density differences are not diffused away. One measure of this is shown in figure 4, which plots the globally averaged T and S from Levitus (1982) observations and from three equilibrium integrations of the global ocean model. The first two are those described above with different eddy parameterizations, which both use restoring boundary conditions at the surface on T and S. The last integration uses equation (24) and the bulk forcing described in Large *et al.* (1997). The globally averaged potential temperature in the horizontal tracer mixing integration is 5.7°C, which is much warmer than the Levitus value of 3.7°C. Using (24) improves this considerably to a value of 3.5°C. Note that the globally averaged

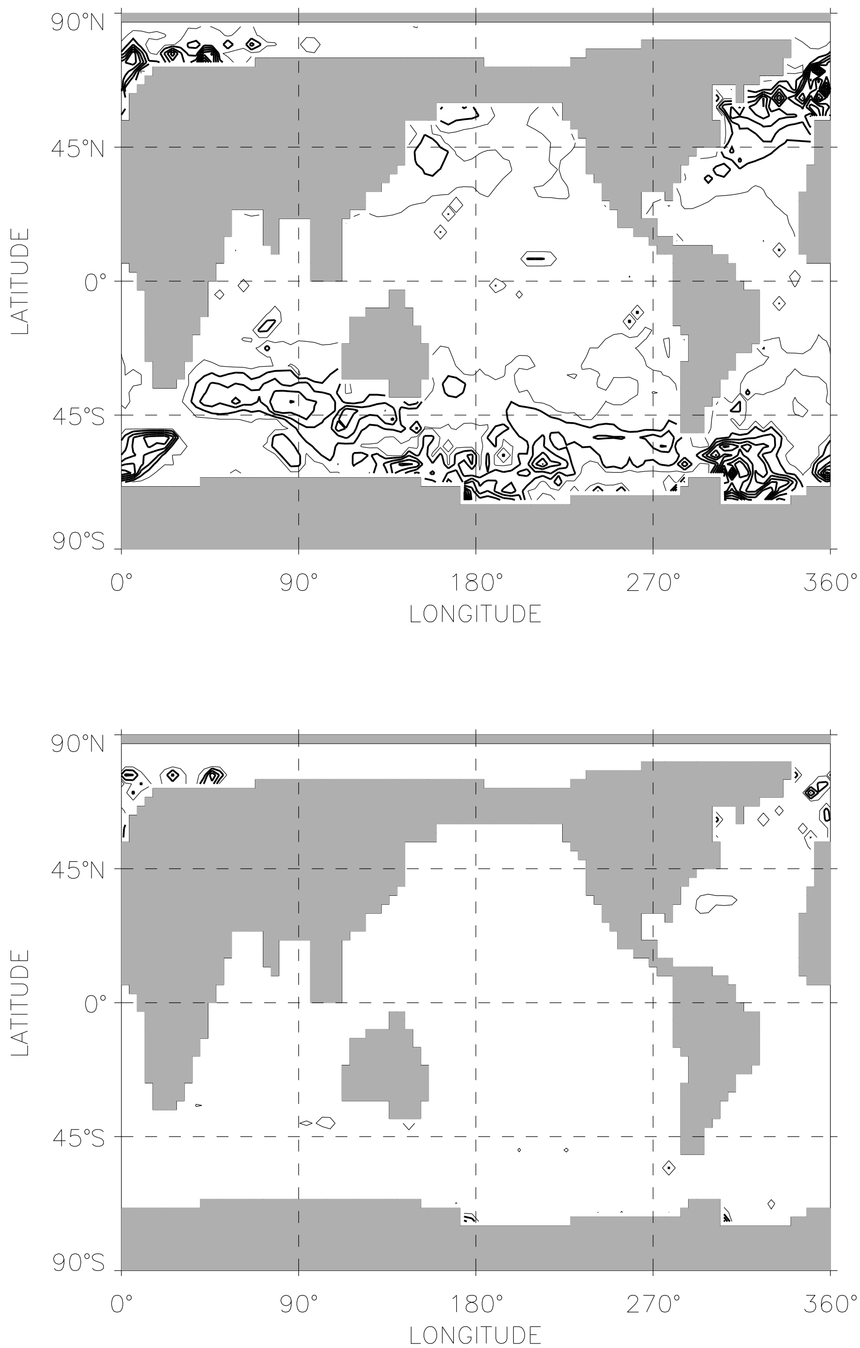

Figure 3. The areas of deep water formation from equilibrium solutions using a) horizontal tracer mixing, and b) using equation (24).

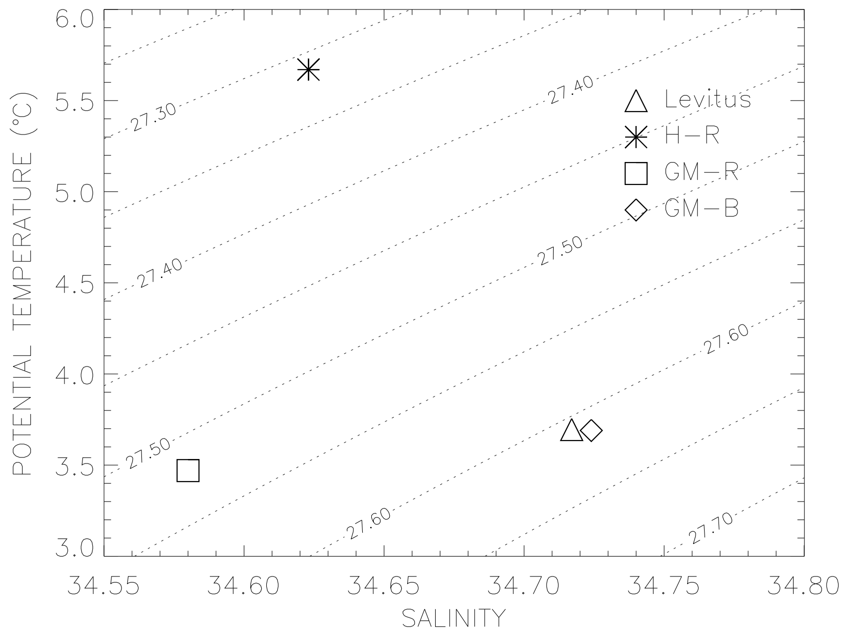

Figure 4. Values of globally-averaged potential temperature and salinity from observations and three equilibrium model solutions: H-R - horizontal tracer mixing and restoring boundary conditions; GM-R - equation (24) and restoring boundary conditions; GM-B - equation (24) and bulk forcing boundary conditions.

salinity changes much less, and remains too fresh compared to reality. This salinity bias is due to the restoring boundary condition on S, which gives a poor representation of the ocean surface fresh water flux. This bias is removed by going to the bulk forcing, which uses realistic precipitation and evaporation fields. The change to bulk forcing affects the globally-averaged potential temperature much less, but does improve it slightly. The values from the third experiment are very close to the Levitus values of 3.7°C and 34.72 ppt.

7. Conclusions

The parameterization of Gent and McWilliams (1990) is shown to be a generalization of the residual-mean circulation to three dimensions. Parameterizing the resulting Eliassen-Palm fluxes determines the momentum equation to use in non-eddy-resolving ocean models. The nonacceleration theorem of Andrews and McIntyre (1976) does not apply exactly to the ocean, but strongly suggests that the eddy-induced advection of tracers should oppose mean flow advection at the latitude of Drake Passage. Results from coarse resolution global ocean models confirm that considerable cancellation does occur at this latitude near the Antarctic Circumpolar Current when the Gent and McWilliams (1990) parameterization is used. This leads to a much more realistic pattern of deep water formation compared to results obtained using the earlier eddy parameterization of horizontal diffusion of tracers.

8. Limerick

There once was an ocean model called MOM,
That occasionally used to bomb,
But eddy advection, and much less convection,
Turned it into a stable NCOM.

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